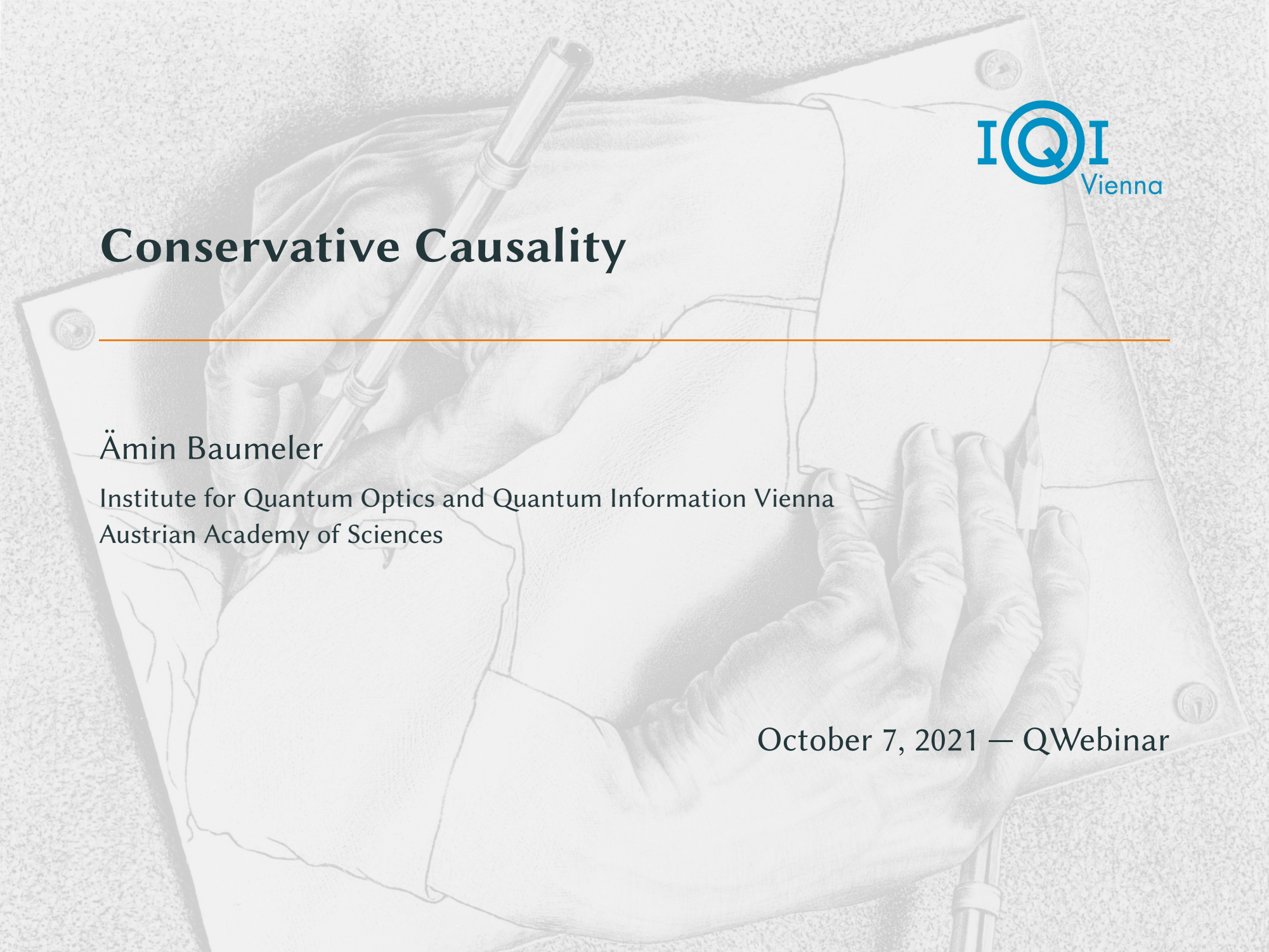
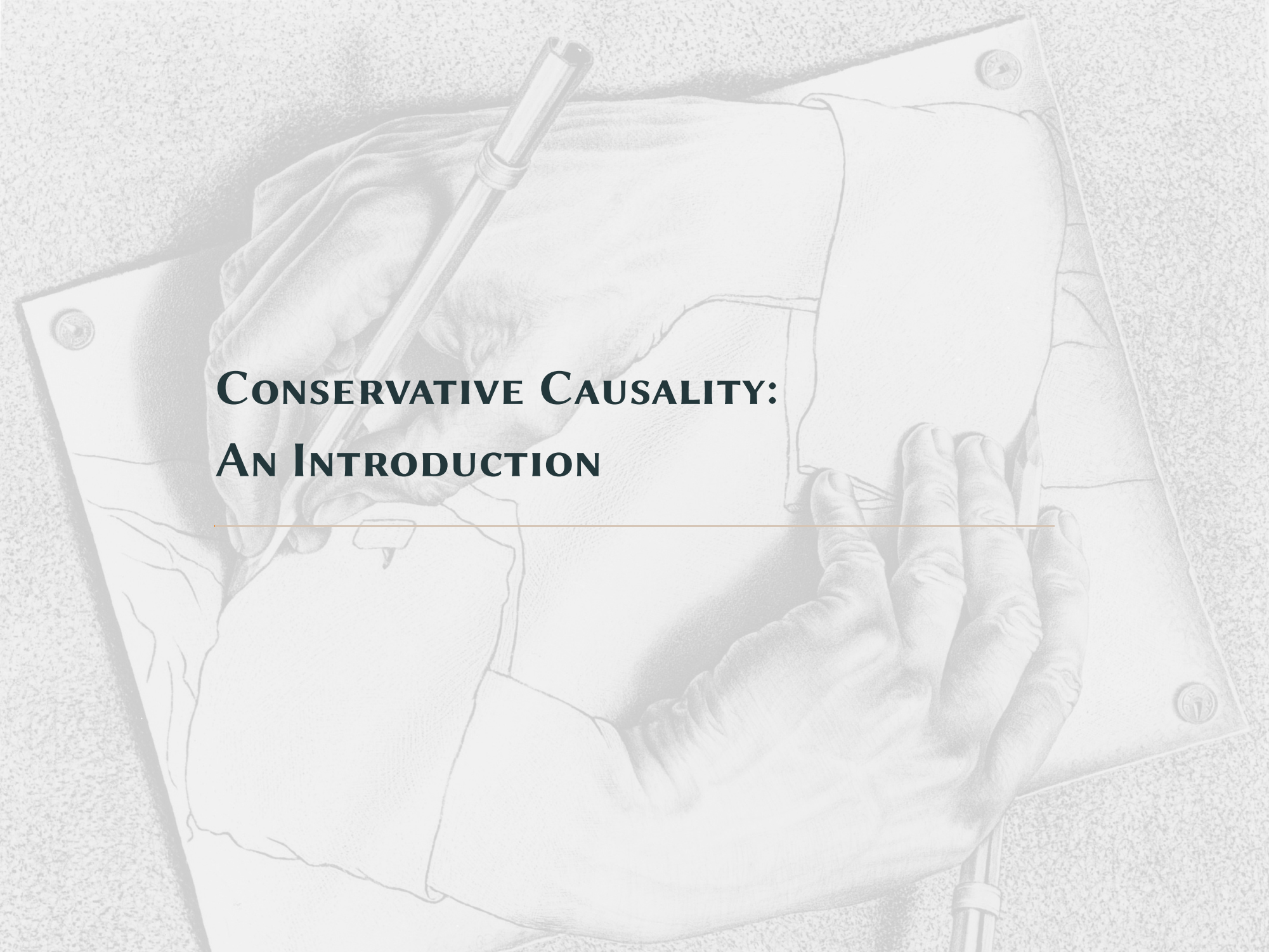


Conservative Causality

Ämin Baumeler

Institute for Quantum Optics and Quantum Information Vienna
Austrian Academy of Sciences

October 7, 2021 — QWebinar



Conservative Causality: An Introduction

Conservative Causality

Comprehensive framework to generalize existing theories while the local description is *conserved*.

Theories of conservative causality lead to a relaxed notion of causality without altering other features.

Conservative Causality: Recipe

1. Take some theory T (*e.g.*, quantum theory, probability theory).
2. Assume that *locally* no deviation from T is observable.
3. Derive global dynamics in the new theory.

Theories of conservative causality have been developed where T is

- quantum theory,
- probability theory,
- discrete functions.

Similarity with General/Special Relativity

$$\frac{\text{Conservative Causality}(T)}{T} \approx \frac{\text{GR}}{\text{SR}}$$

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012); ÄB, S Wolf, NJP 18:1 (2016); ÄB, S Wolf, NJP 18:3 (2016)

Conservative Causality: Motivations

Motivations

- *What is the origin of causal order?*
- *What are the consequences of violating causal order?*
- *How can we combine operational theories with general relativity?*

What is the origin of causal order?

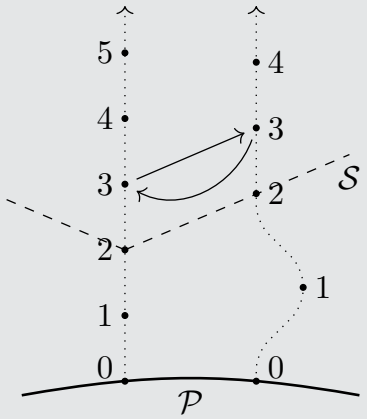
Conservative causality allows us to search for a principle from which causal order follows.

Relevance

- Wheeler's puzzle:

  "How to derive time without presupposing time."

- Are closed time-like curves mathematical artefacts of general relativity?



J A Wheeler, IBM Journal of Research and Development 32 (1988); F Echeverria, G Klinkhammer, K S Thorne, PRD 44 (1991)

What are the consequences of violating causal order?

Conservative causality allows us to study enhanced information processing beyond causal order.

Relevance

- How does causal order *restrict* information processing?
- Derive new bounds for information processing.
- What is the foundational status of causal order for *e.g.*, Bell non-local correlations?

How can we combine operational theories with general relativity?

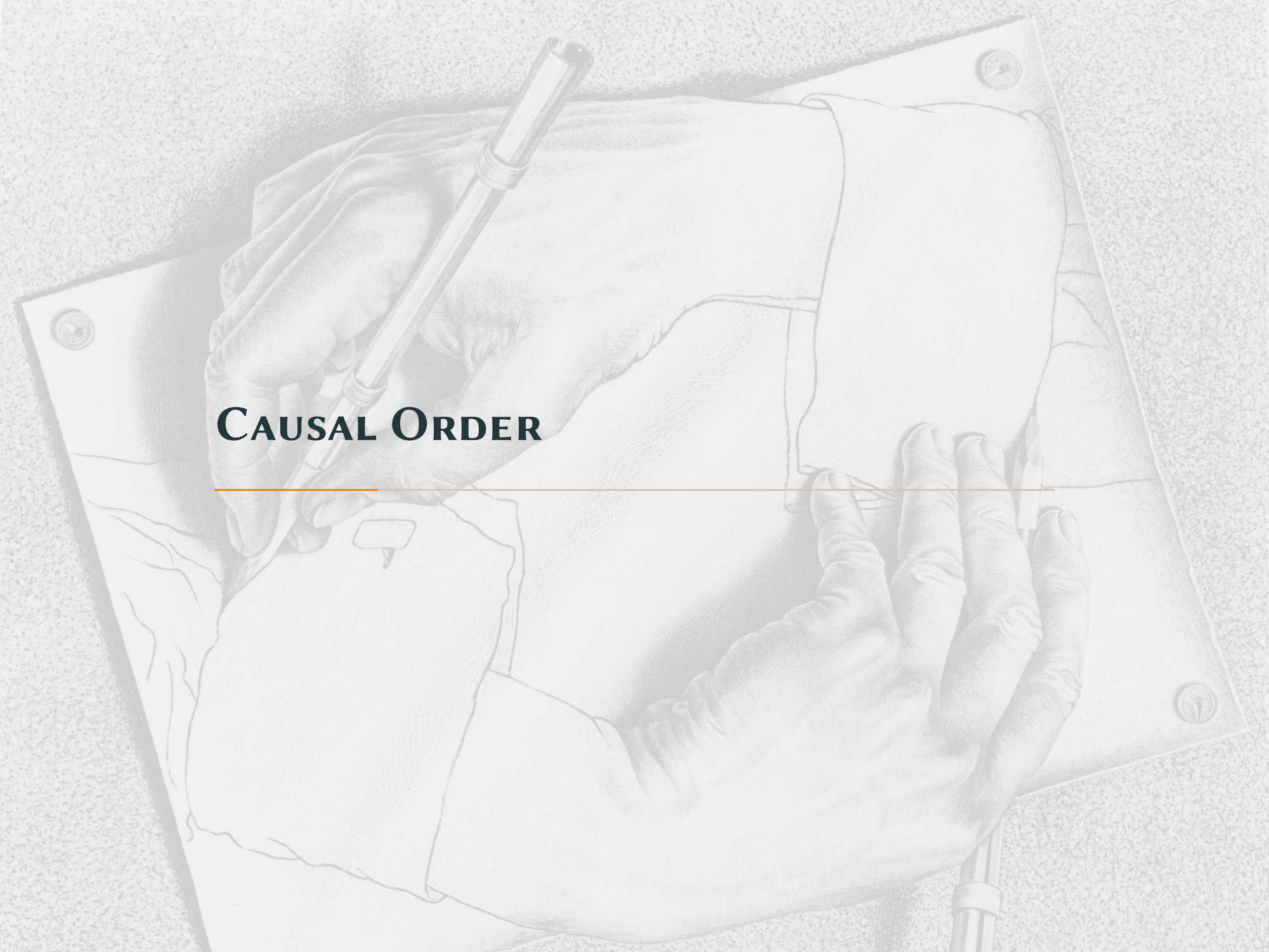
Conservative causality gives toy theories that resemble general relativity:

- Special relativity holds for free-falling observers in general relativity
- A theory T holds for observers in the theory of conservative causality constructed with T

Relevance

- Predict dynamics in quantum gravity.
- Conservative causality is simpler (less physical content, *e.g.*, no ED, no fields) and more general (T can be any theory).

Overview

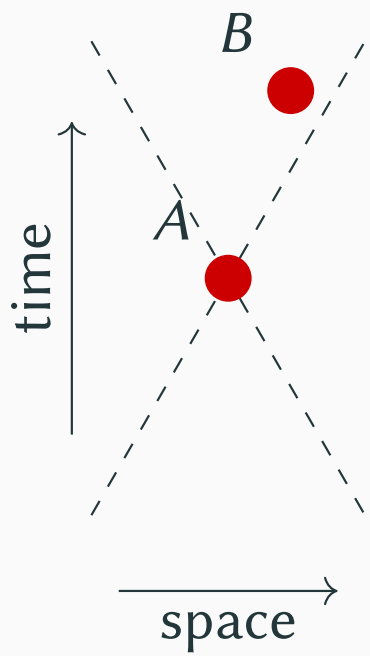


Causal Order

Causal Order

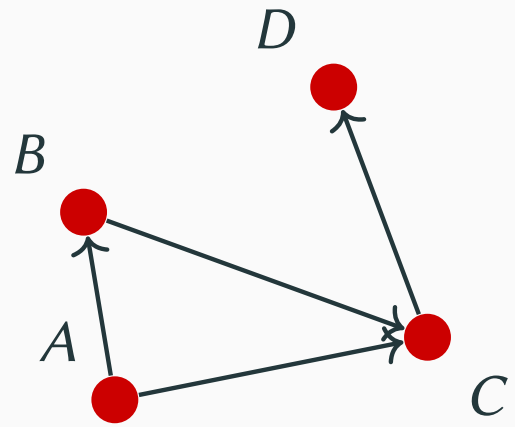
Causal order: *No event is influenced by its future.*

Space-time diagram



No worldline from B to A.

Causal structure

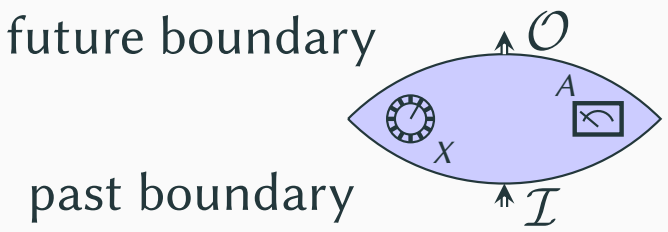


The causal structure is given by a *directed acyclic graph* with model parameters, P_A, $P_{B|A}$, $P_{C|AB}$, $P_{D|C}$.

J Pearl, "Causality" (Cambridge University Press, 2009)

Causal Order: An Operational Approach

Party: localized space-time region; can do any *intervention* on the system received

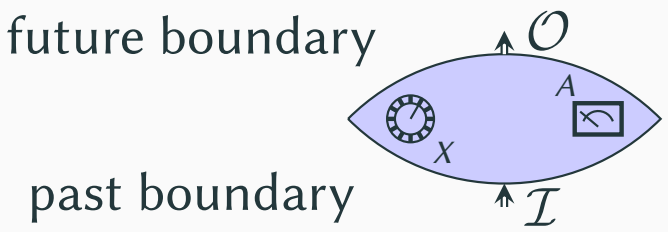


Every party has

- a setting (R.V.; here X),
- a result (R.V.; here A),
- an input, and an output.

Causal Order: An Operational Approach

Party: localized space-time region; can do any *intervention* on the system received



Every party has

- a setting (R.V.; here X),
- a result (R.V.; here A),
- an input, and an output.

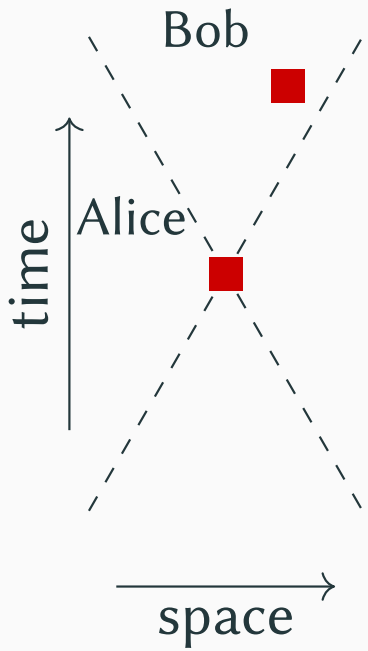
Example

$\mathcal{I}, \mathcal{O}$ are random variables; the intervention is a stochastic map $P_{AO|XI}$.

Causal Order: An Operational Approach

Event: *A single experiment of a party; a single intervention.*

Causal order: *No party is influenced by future parties.*



Alice: setting X, result A

Bob: setting Y, result B

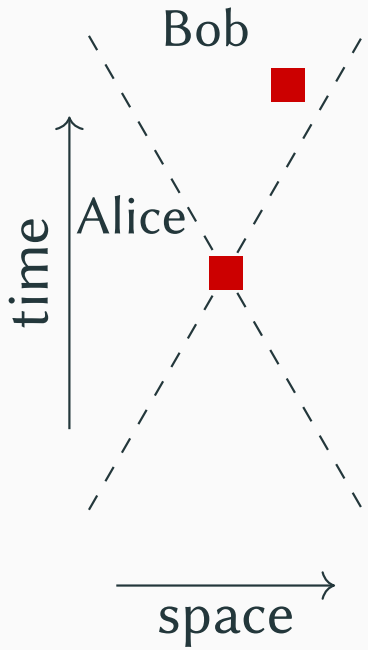
Possible correlations between Alice and Bob:

$$P_{AB|XY} = P_{A|X} P_{B|AXY} \,.$$

Causal Order: An Operational Approach

Event: *A single experiment of a party; a single intervention.*

Causal order: *No party is influenced by future parties.*



Alice: setting X, result A

Bob: setting Y, result B

Possible correlations between Alice and Bob:

$$P_{AB|XY} = P_{A|X} P_{B|AXY} \,.$$

Note

This is *independent* of the underlying theory.

Causal Order: Two-Party Case

Definition (Two-party causal correlations)

Two-party correlations $P_{AB|XY}$ are *causal* if and only if they can be decomposed as

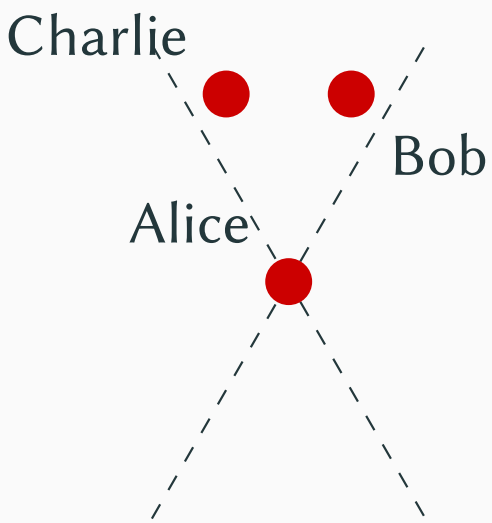
$$P_{AB|XY} = \lambda P_{A|X} P_{B|AXY} + (1-\lambda) P_{B|Y} P_{A|BXY} \,,$$

for some $0 \leq \lambda \leq 1$.

So, these are correlations that are *at most* one-way signalling.

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

Causal Order: Multi-Party Case



Since Alice can control her future, she can also control the causal relation between Bob and Charlie.
This can be done, in general relativity, *e.g.*, by moving a mass.

ÄB, S Wolf, ISIT (IEEE 2014); A Abbot *et al.*, PRA 94 (2016)

Causal Order: Multi-Party Case

Definition (Multi-party causal correlations)

N-party correlations $P_{A_1A_2\cdots|X_1X_2\ldots}$ are *causal* if and only if they can be decomposed as

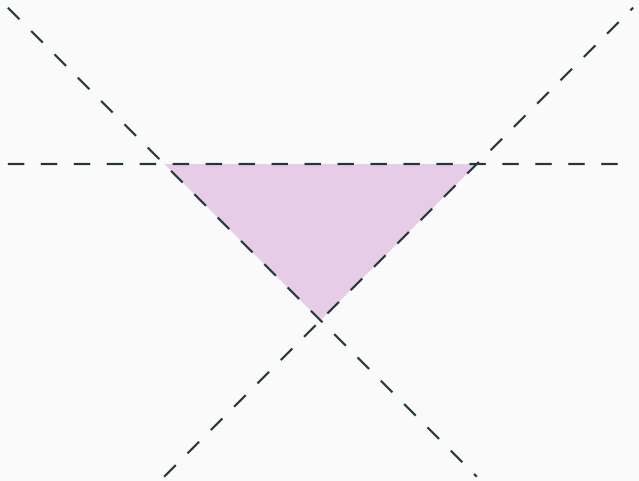
$$P_{A_1A_2\cdots|X_1X_2\ldots} = \sum_{k=1}^{N} \lambda_k P_{A_k|X_k} P_{A_{\setminus k}|A_kX} \,,$$

for some $0 \leq \lambda_k \leq 1$ with $\sum_{k=1}^{N} \lambda_k = 1$, and where $P_{A_{\setminus k}|A_kX}$ are $(N-1)$-party causal correlation.

ÄB, S Wolf, ISIT (IEEE 2014); A Abbot *et al.,* PRA 94 (2016)

Causal Order: Polytopes

This definition of causal order gives a polytope of N-party correlations $P_{A_1A_2\cdots|X_1X_2\ldots}$.



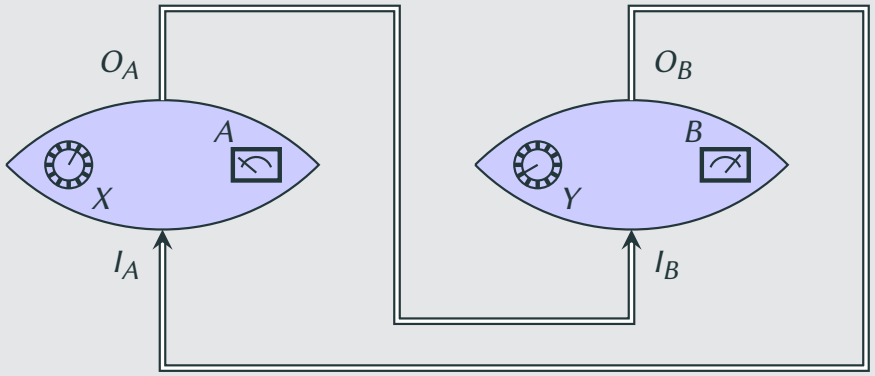
Correlations that lie *outside* of this polytope are called *non-causal.*

Extreme Departures From Causal Order

A motivation to uphold causal order is logical consistency!

Grandfather paradox

Imagine a two-party setup where each party influences the other, *e.g.*, like this:



where the connections represent identity channels, and where the inputs/outputs are single bits.

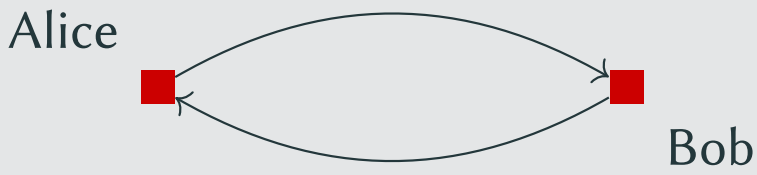
Then, if Alice forwards her input bit ($O_A = I_A$), and Bob negates it ($O_B = I_B \oplus 1$), we reach a contradiction!

Extreme Departures From Causal Order

A motivation to uphold causal order is logical consistency!

Grandfather paradox

Imagine a two-party setup where each party influences the other, *e.g.*, like this:



where the connections represent identity channels, and where the inputs/outputs are single bits.

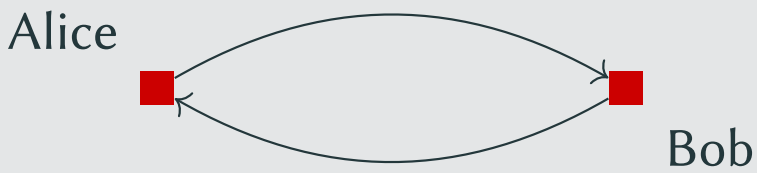
Then, if Alice forwards her input bit ($O_A = I_A$), and Bob negates it ($O_B = I_B \oplus 1$), we reach a contradiction!

Extreme Departures From Causal Order

A motivation to uphold causal order is logical consistency!

Information paradox

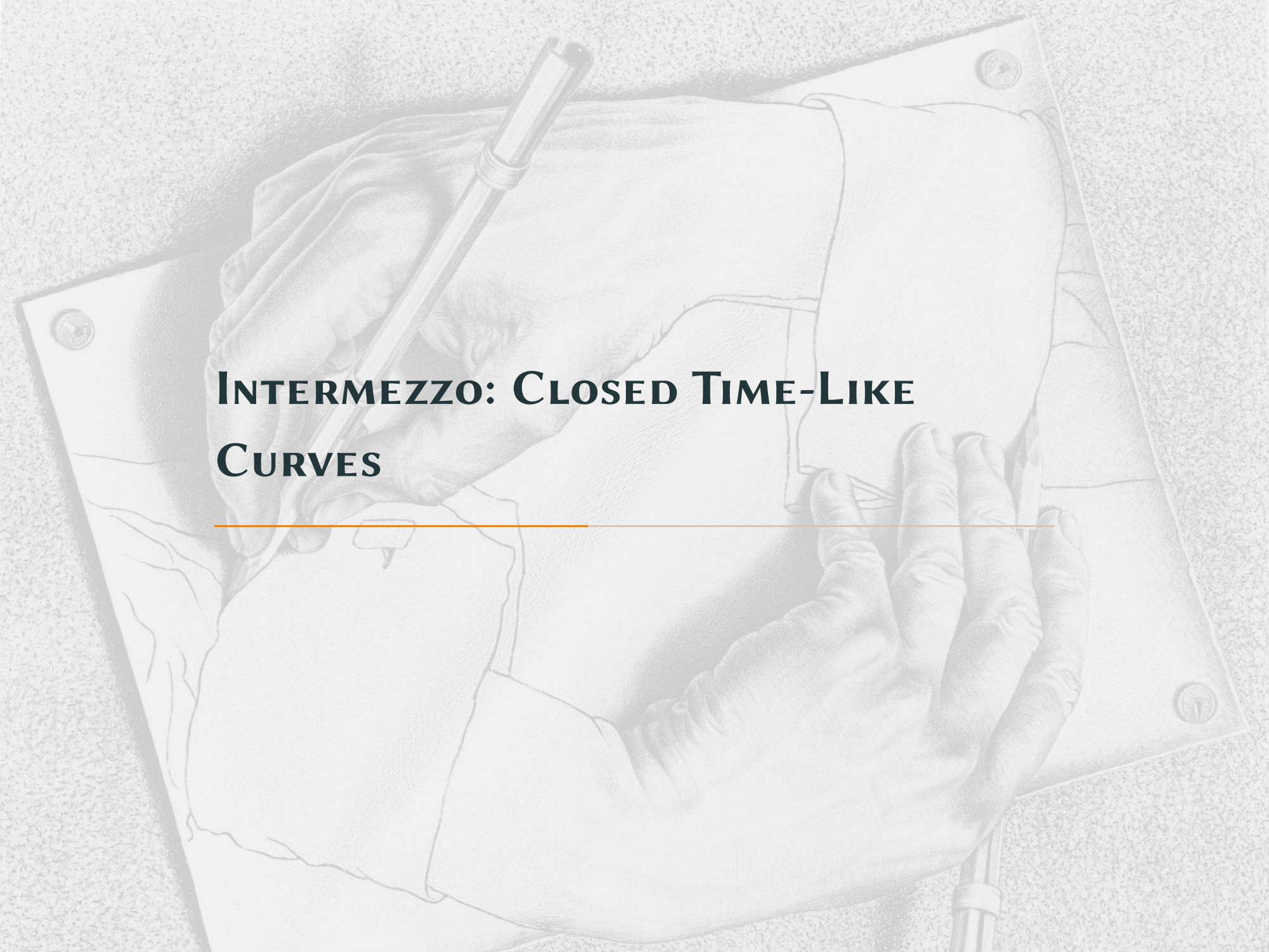
Imagine a two-party setup where each party influences the other, *e.g.*, like this:



where the connections represent identity channels, and where the inputs/outputs are single bits.

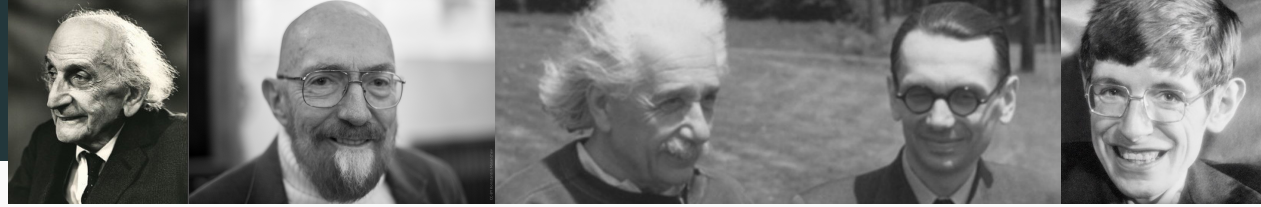
Then, if Alice forwards her input bit ($O_A = I_A$), and Bob as well ($O_B = I_B$), we have two consistent solutions!

Overview



Intermezzo: Closed Time-Like Curves

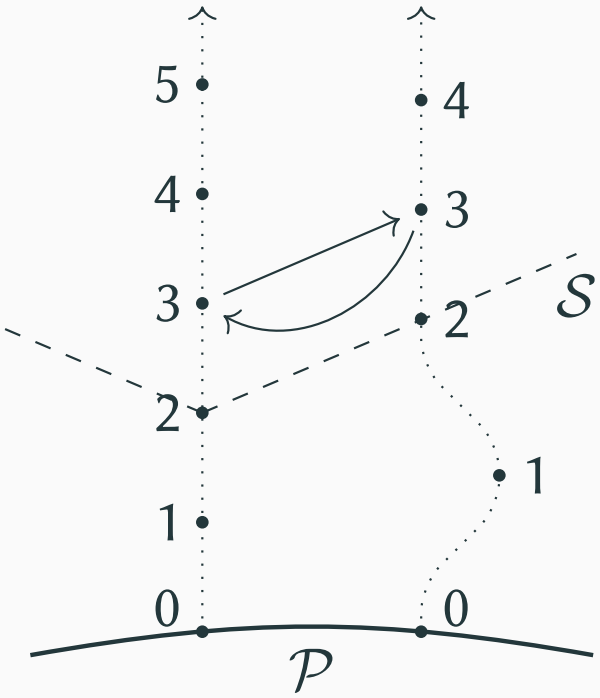
On CTCs



- Einstein predicted in 1914 that GR allows for CTCs.
- Since 1924, solutions to the Einstein equations with CTCs are knwon.
- CTCs are self-consistent (*).
- However, the latest studies show that they lead to a “Billiard-Ball Crisis.”
- Hawking formulated in 1991 the “Chronology Protection Conjecture.”

A Einstein, Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften 2 (1914); K Lanczos, Zeitschrift für Physik 21 (1924); K Gödel, Rev Mod Phys 21 (1949); F Echeverria, G Klinkhammer, K S Thorne, PRD 44 (1991); S Hawking, PRD 46 (1991)

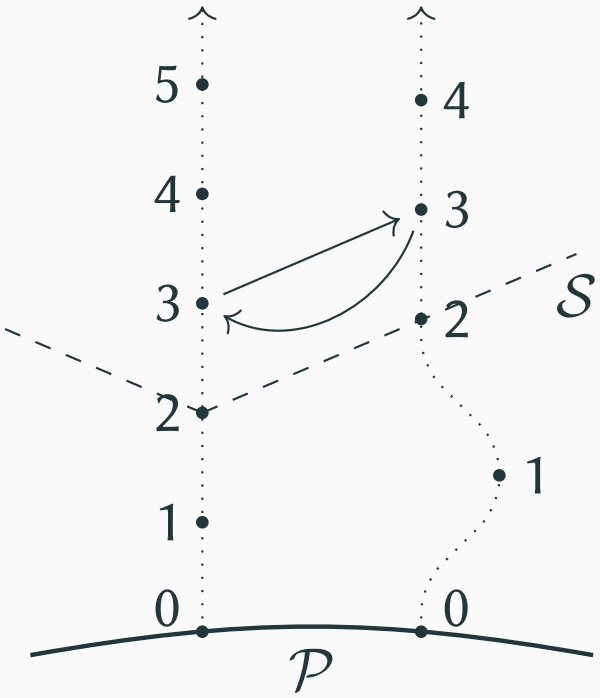
Self-Consistency and the Billiard-Ball Crisis



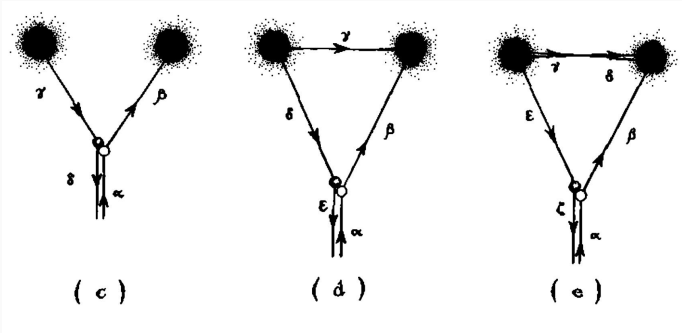
Novikov's self-consistency principle: Only consistent dynamics *without* altering physics.

I Novikov, J Exp Th Phys 68 (1989); J Friedman *et al.,* PRD 42 (1990); K Thorne, "Black holes and time warps" (W W Norton, 1994)

Self-Consistency and the Billiard-Ball Crisis

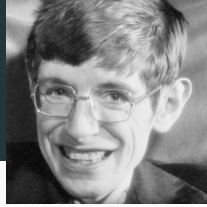


Not zero, but *infinitely many* solutions!



I Novikov, J Exp Th Phys 68 (1989); J Friedman *et al.,* PRD 42 (1990); K Thorne, "Black holes and time warps" (W W Norton, 1994)

Chronology Protection Conjecture



Based on physical energy conditions, one can rule out some forms of closed time-like curves that arise in general relativity.

PHYSICAL REVIEW D VOLUME 46, NUMBER 2 15 JULY

Chronology protection conjecture

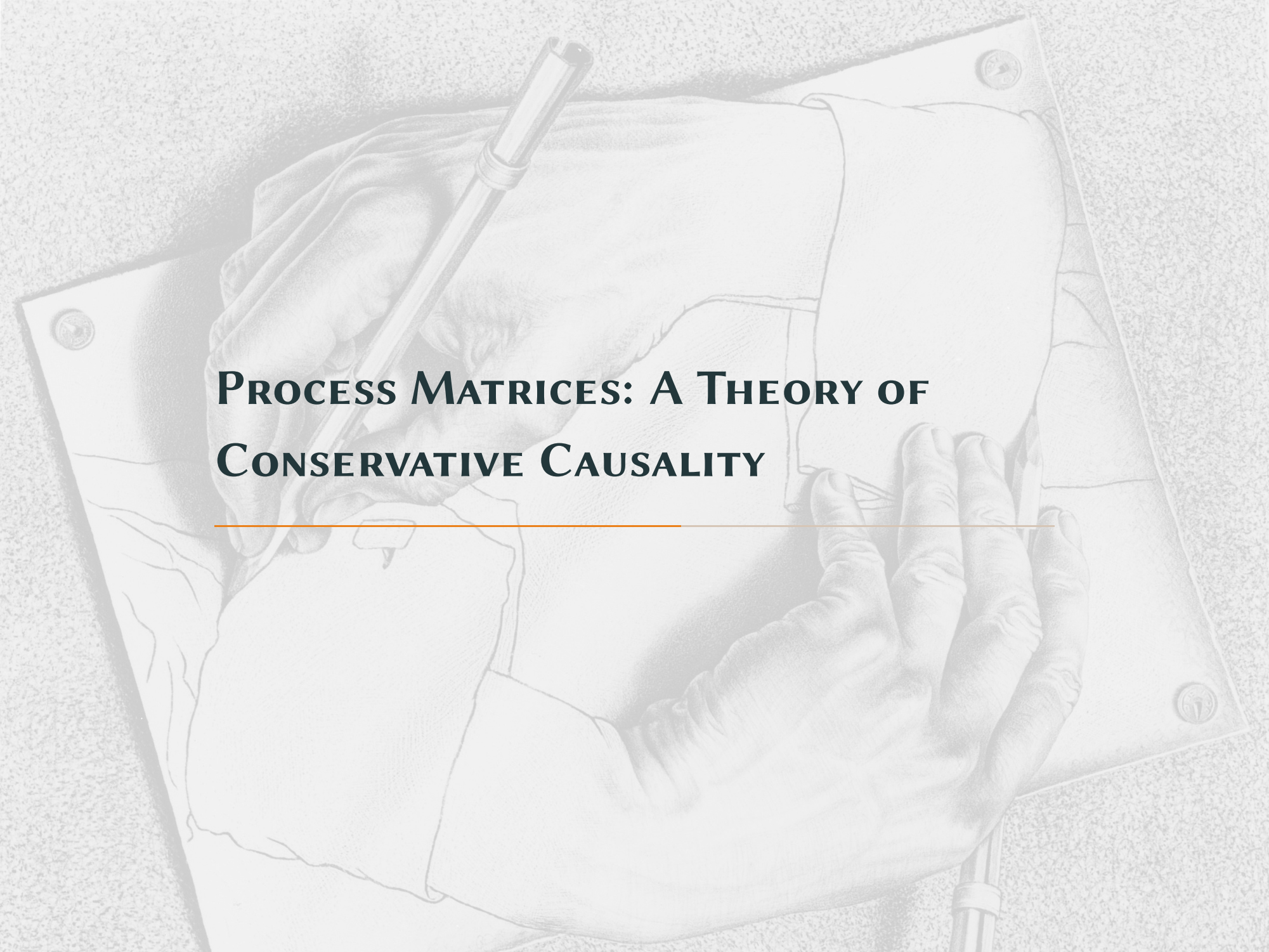
S. W. Hawking
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge CB3 9EW, United Kingdom
(Received 23 September 1991)

But: We do not have a complete theory of physics. Many missing points: Quantum gravity, black holes, the minuscule “quantum foam.”

Are there other arguments?

S. Hawking, PRD 46 (1991)

Overview



Process Matrices: A Theory of Conservative Causality

The Process-Matrix Framework

Assumptions

1. Every party performs a single quantum experiment of her choice (intervention)
2. Parties are isolated (and can share arbitrary quantum systems)
3. Locally, no deviation from quantum theory is observable

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

The Process-Matrix Framework

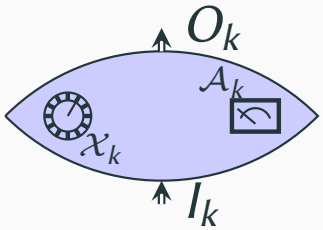
Assumptions

1. Every party performs a single quantum experiment of her choice (intervention)
2. Parties are isolated (and can share arbitrary quantum systems)
3. Locally, no deviation from quantum theory is observable

Every party k is described by a quantum instrument:

$$\left\{\mu^k_{a_k|x_k} : \mathcal{L}(\mathcal{H}_{I_k}) \to \mathcal{L}(\mathcal{H}_{O_k})\right\}_{(a_k,x_k)\in\mathcal{A}_k\times\mathcal{X}_k}$$

$$\forall x_k \in \mathcal{X}_k : \sum_{a_k\in\mathcal{A}_k} \mu^k_{a_k|x_k} \text{ is CPTP}$$



O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

The Process-Matrix Framework

Assumptions

1. Every party performs a single quantum experiment of her choice (intervention)
2. Parties are isolated (and can share arbitrary quantum systems)
3. Locally, no deviation from quantum theory is observable

The joint description of N parties is given by the collection:

$$\bigtimes_{k=1}^{N} \mu^{k}_{a_k|x_k}$$

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

The Process-Matrix Framework

Assumptions

1. Every party performs a single quantum experiment of her choice (intervention)
2. Parties are isolated (and can share arbitrary quantum systems)
3. Locally, no deviation from quantum theory is observable

The observations of the parties are given by *multi-linear* functions ω:

$$\forall \bigtimes_{k=1}^{N} \mu^{k}_{a_k|x_k} \quad \exists P_{A_1,\ldots,A_N|X_1,\ldots,X_N} :$$

$$\omega\Big(\bigtimes_{k=1}^{N} \mu^{k}_{a_k|x_k}\Big) = P_{A_1,\ldots,A_N|X_1,\ldots,X_N}(a_1,\ldots,a_N \mid x_1,\ldots,x_N)$$

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

The Process-Matrix Framework

The description is greatly simplified if the quantum instruments are expressed in the Choi-Jamiołkowski picture:

$$\mu^k_{a_k|x_k} \leftrightarrow M^k_{a_k|x_k} \in \mathcal{L}(\mathcal{H}_{I_k} \otimes \mathcal{H}_{O_k})$$

$$\omega \leftrightarrow W \in \mathcal{L}\Big(\bigotimes_{k=1}^{N} \mathcal{H}_{I_k} \otimes \mathcal{H}_{O_k}\Big)$$

$$\omega\Big(\bigtimes_{k=1}^{N} \mu^k_{a_k|x_k}\Big) = \mathrm{Tr}\left[\Big(\bigotimes_{k=1}^{N} M^k_{a_k|x_k}\Big) W\right]$$

Definition (Process matrix)

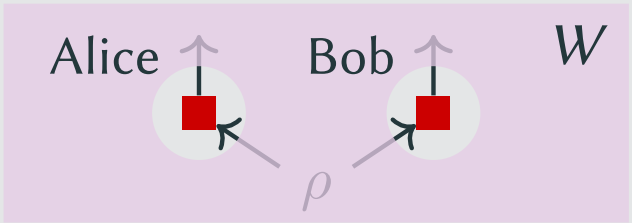
The operator W is a *process matrix* if and only if for all quantum instruments $\mathrm{Tr}\left[\left(\bigotimes_{k=1}^{N} M^k_{a_k|x_k}\right) W\right]$ is a probability distribution $P_{A_1,\ldots,A_N|X_1,\ldots,X_N}$.

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012); A Jamiołkowski, Rep Math Phys 3 (1972); M D Choi, Lin Alg Appl 10 (1975)

The Process-Matrix Framework: Discussion

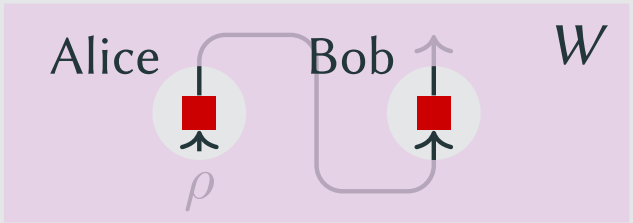
A process matrix W is a *resource* that establishes the correlations among the parites. It unifies states and channels.

Shared state



$$W = \rho_{I_A,I_B} \otimes \mathbb{1}_{O_A,O_B}$$

Communication channel



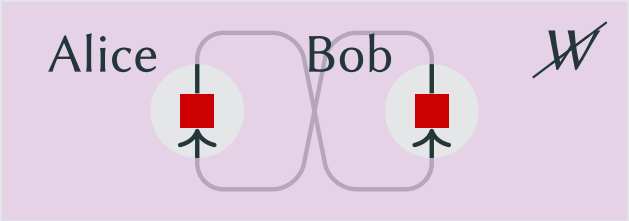
$$W = \rho_{I_A} \otimes |\mathbb{1}\rangle\rangle\langle\langle\mathbb{1}|_{O_A,I_B} \otimes \mathbb{1}_{O_B}$$

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

The Process-Matrix Framework: Discussion

A process matrix W is a *resource* that establishes the correlations among the parites. It unifies states and channels.

Non-example: Loop



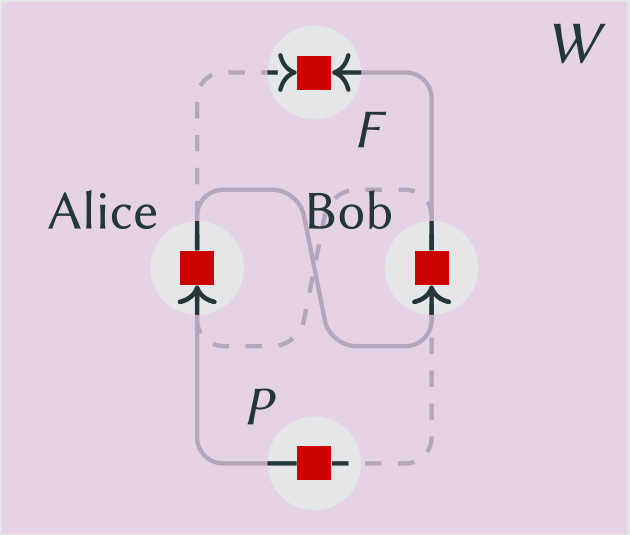
$$\not{W} = |\mathbb{1}\rangle\rangle\langle\langle\mathbb{1}|_{O_B,I_A} \otimes |\mathbb{1}\rangle\rangle\langle\langle\mathbb{1}|_{O_A,I_B}$$

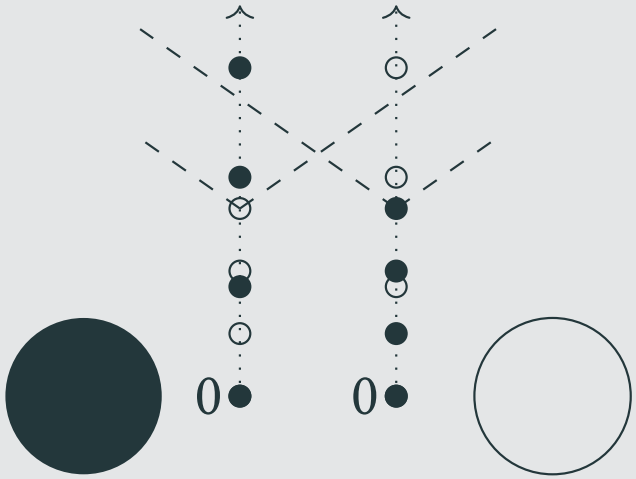
$$\exists \text{CPTP maps } M^{\text{Alice}}, M^{\text{Bob}} : \text{Tr}[M^{\text{Alice}} \otimes M^{\text{Bob}} \not{W}] = 0$$

O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012)

The Process-Matrix Framework: General-relativistic dynamics

The Quantum Switch

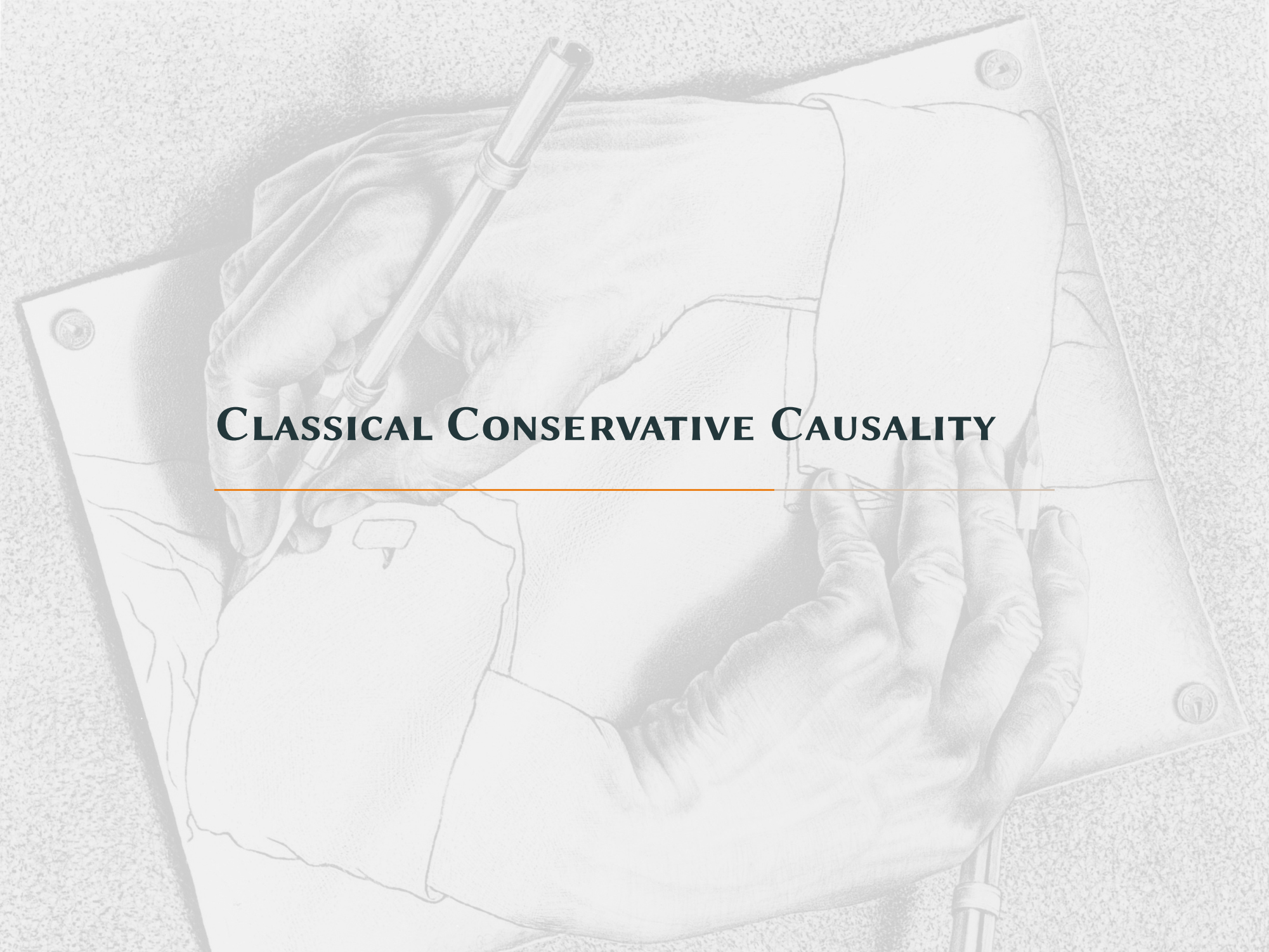




$$W = |w\rangle\rangle\langle\langle w|$$

$$|w\rangle\rangle = |0\rangle_{P_1}|\mathbb{1}\rangle\rangle_{P_2,I_A}|\mathbb{1}\rangle\rangle_{O_A,I_B}|\mathbb{1}\rangle\rangle_{O_B,F_2}|0\rangle_{F_1} + |1\rangle_{P_1}|\mathbb{1}\rangle\rangle_{P_2,I_B}|\mathbb{1}\rangle\rangle_{O_B,I_A}|\mathbb{1}\rangle\rangle_{O_A,F_2}|1\rangle_{F_1}$$

T Colnaghi *et al.*, Phys Lett A 376 (2012); O Oreshkov, F Costa, Č Brukner, Nat Comm 3 (2012); M Zych *et al.*, Nat Comm 10 (2019)



Classical Conservative Causality

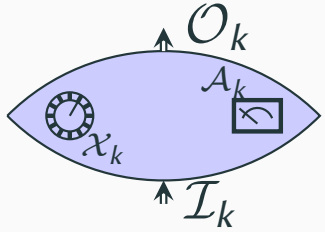
The Process-Function Framework

Assumptions

1. Every party performs a function of her choice (intervention)
2. Parties are isolated
3. Locally, no contradiction is reached

Here, input and output spaces are *discrete* sets. Party k is described by a function:

$$\mu_k : \mathcal{X}_k \times \mathcal{I}_k \to \mathcal{A}_k \times \mathcal{O}_k \,.$$



The Process-Function Framework

Assumptions

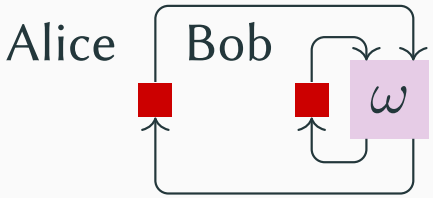
1. Every party performs a function of her choice (intervention)
2. Parties are isolated
3. Locally, no contradiction is reached

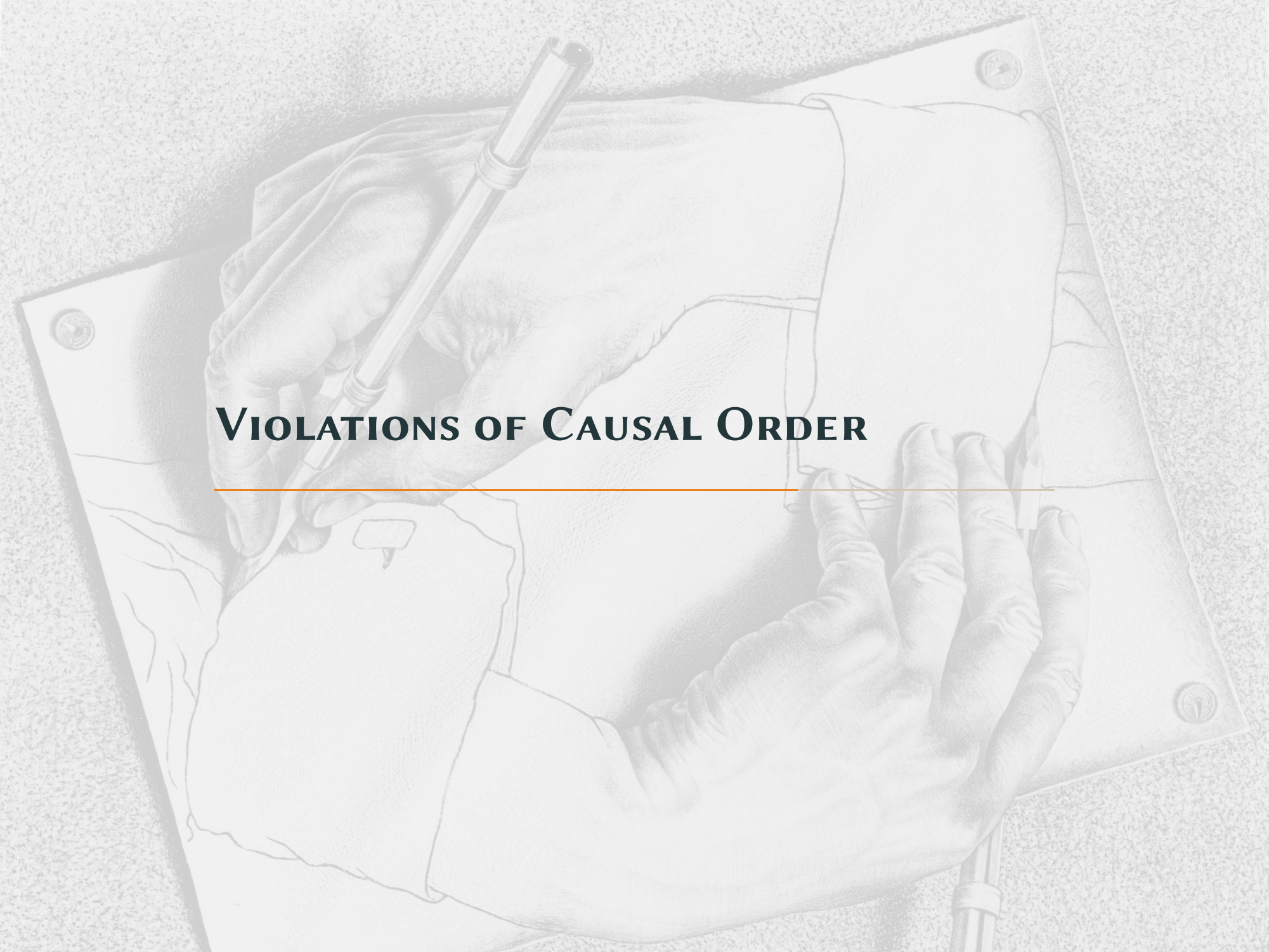
Theorem

A process-function ω is a function

$$\omega : \bigtimes_{k=1}^{N} \mathcal{O}_k \to \bigtimes_{k=1}^{N} \mathcal{I}_k$$

$$\forall \{\mu_k : \mathcal{I}_k \to \mathcal{O}_k\}_{1 \leq k \leq N}, \exists x : x = \omega(\mu(x))$$





Violations of Causal Order

A Causal Inequality

Three-party Game

Every party Alice, Bob, Charlie has a binary setting and a binary result: $\mathcal{X} = \mathcal{Y} = \mathcal{Z} = \mathcal{A} = \mathcal{B} = \mathcal{C} = \{0, 1\}$. Their results must satisfy

$$A = \neg Y \wedge Z\,, \qquad B = \neg Z \wedge X\,, \qquad C = \neg X \wedge Y\,.$$

M Araújo, A Feix (private communication); ÄB, S Wolf, NJP 18:1 (2016)

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Lemma

The maximum winning probability with causal correlations $P_{ABC|XYZ}$ *is* $3/4$.

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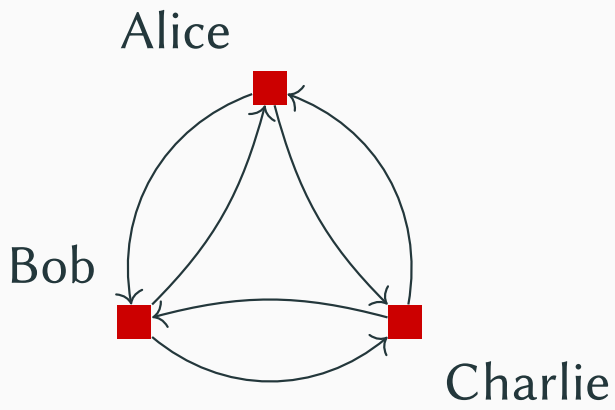
Lemma

There exists a process function ω *such that they win the game* deterministically.

M Araújo, A Feix (private communication); ÄB, S Wolf, NJP 18:1 (2016)

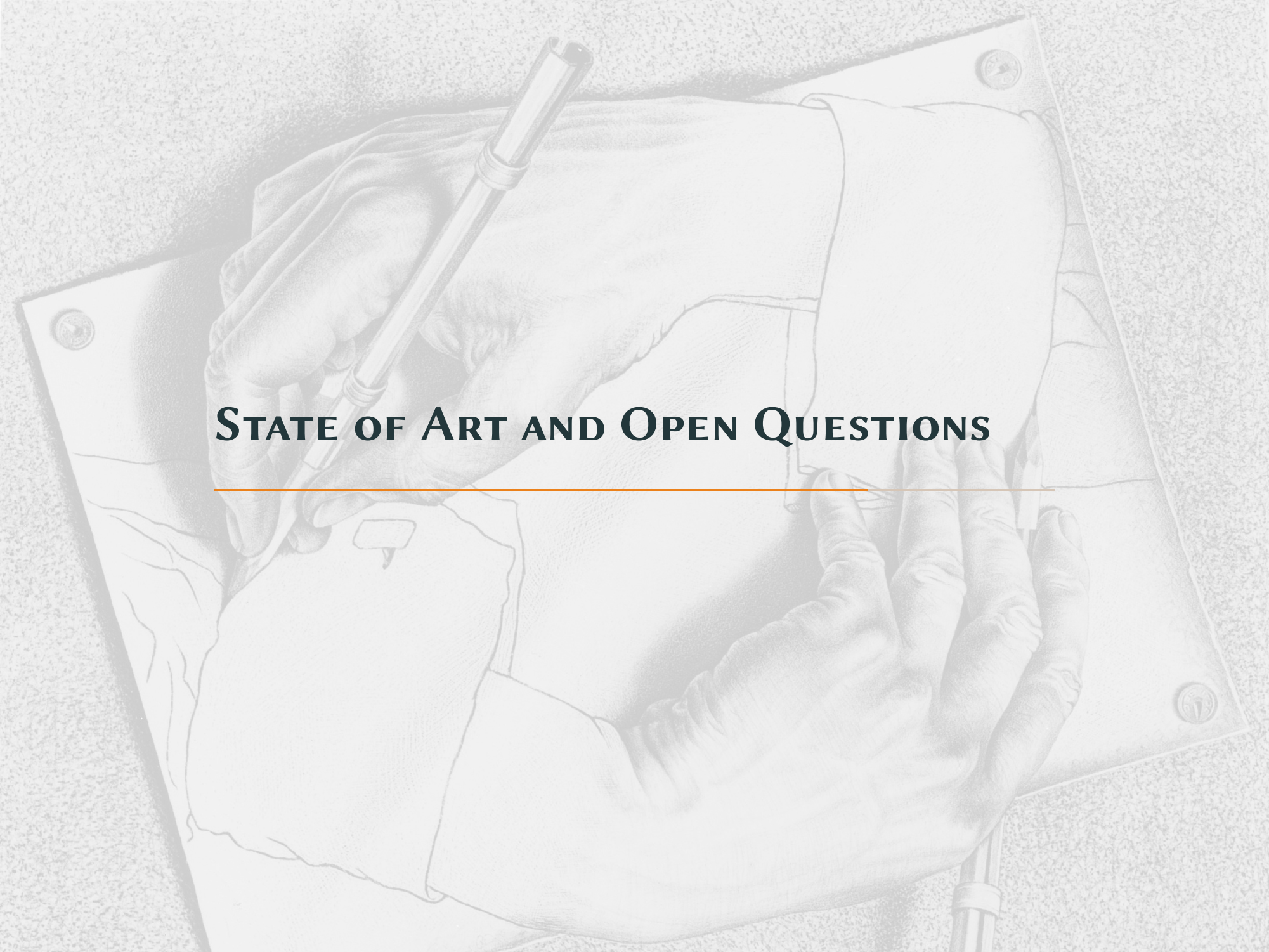
Violation of Causal Inequality

This means: there exist processes with *cyclic* causal structures, *e.g.*,



See also C Branciard *et al.*, NJP 18 (2015) for a collection of simple causal inequalities and their violations.

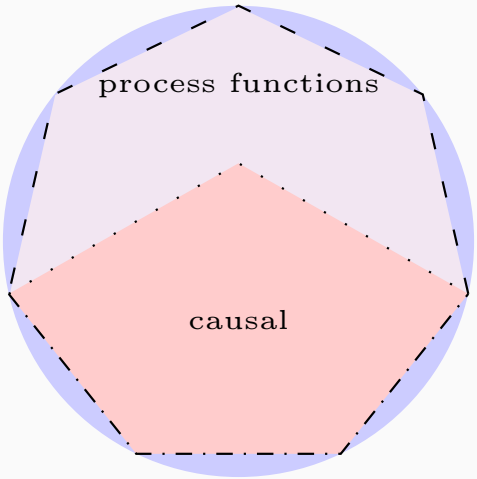
Overview



State of Art and Open Questions

Conservative Causality: Properties

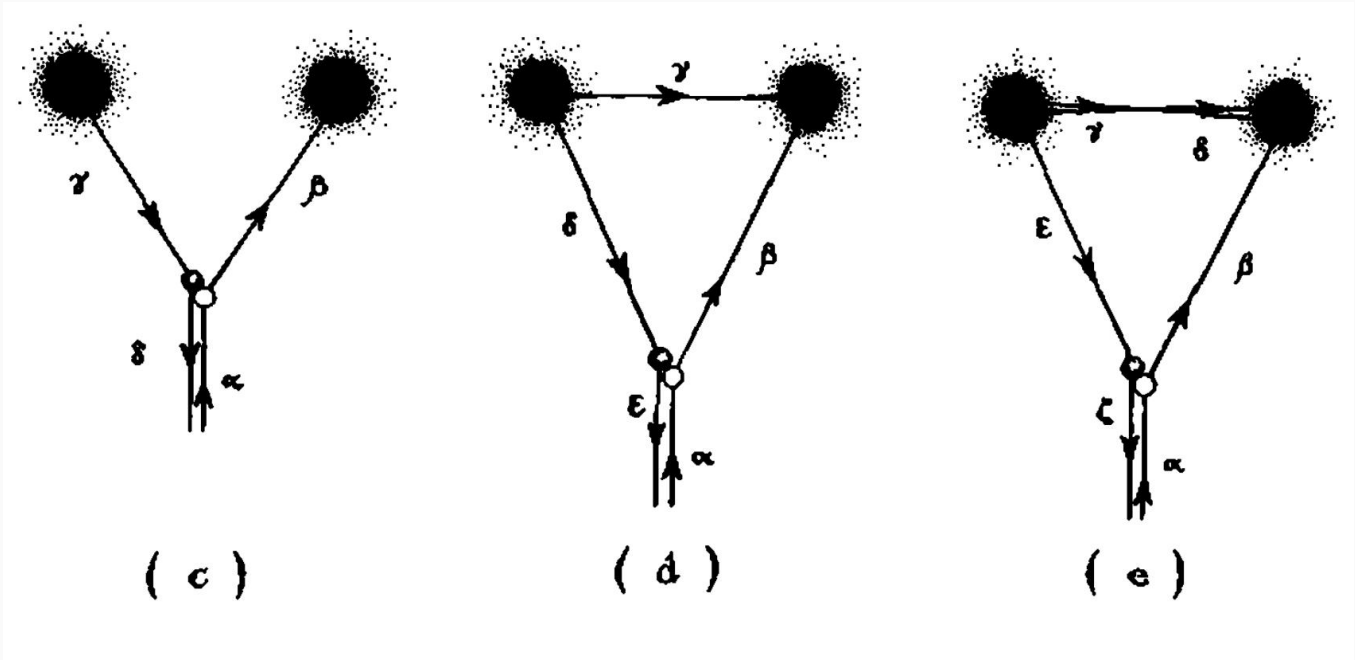
Deterministic violations of causal order for any number of parties.



ÄB, A S Gilani, J Rashid, arXiv:2104.06234 [quant-ph] (2021)

Conservative Causality: Properties

No Billiard-Ball Crisis: Equivalence of grandfather and information paradox.



ÄB, E Tselentis, in proceedings QPL 2020, EPTCS 340 (2021)

Conservative Causality: Properties

Enhanced information-processing with the quantum switch.

- Query complexity: Quadratic advantage over causal quantum protocols for channel discrimination
- Communication complexity: Possibly exponential advantage over causal quantum protocols
- Communication through channels with zero information capacity
- Query complexity: Probabilistically inverting unitary operations

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G Chiribella *et al.,* PRA 88 (2013); P Allard Guerin *et al.,* PRL 117 (2016); D Ebler, S Salek, G Chiribella, PRL 120 (2018); M Quintino *et al.,* PRL 123 (2019); M Renner, Č Brukner, arXiv:2102.11293 [quant-ph] (2021)

Conservative Causality: Properties

Process functions cannot solve NP-hard problems efficiently.
Process matries cannot solve PP-hard problems efficiently.

The quantum bound (process matrices) is believed to be loose: It follows from the model of postselected CTCs.

Note

Other models of CTCs solve PP-complete or
even PSPACE-complete problems in polynomial time.

ÄB, S Wolf, PRSA 474 (2018); M Araújo, P Allard Guerin, ÄB, PRA 96 (2017)

Conservative Causality: Properties

All dynamics are linear and reversible.

More precisely, the framework is by definition linear.

Demanding reversibility does not rule out any of the other features.

ÄB, S Wolf, NJP 18:1 (2016); ÄB, S Wolf, NJP 18:3 (2016); M Araújo *et al.,* Quantum 1 (2017)

Conservative Causality: Properties

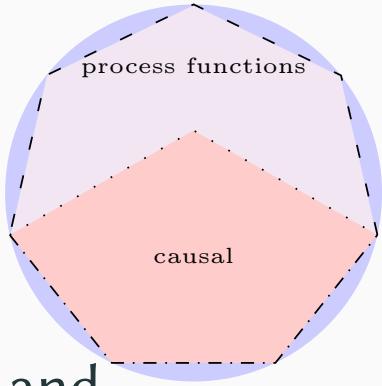
Processes can be understood as closed time-like curves.

Conservative causality models closed time-like curves (CTCs).
These CTCs are tame in every sense mentioned before:

- No Billiard-Ball Crisis
- Reversible and deterministic
- Computationally tame

ÄB *et al.*, Class Quant Grav 36 (2019)

Conservative Causality: Properties



- Deterministic violations of causal order for any number of parties.
- No Billiard-Ball Crisis: Equivalence of grandfather and information paradox.
- Enhanced information-processing with the quantum switch.
- Process-functions cannot solve NP-hard problems efficiently.
- Process-matries cannot solve PP-hard problems efficiently (*).
- All dynamics are linear and reversible.
- Processes can be understood as closed time-like curves.

ÄB, A S Gilani, J Rashid, arXiv:2104.06234 [quant-ph] (2021); ÄB, E Tselentis, in proceedings QPL 2020, EPTCS 340 (2021); G Chiribella *et al.,* PRA 88 (2013); P Allard Guerin *et al.,* PRL 117 (2016); D Ebler, S Salek, G Chiribella, PRL 120 (2018); M Quintino *et al.,* PRL 123 (2019); M Renner, Č Brukner, arXiv:2102.11293 [quant-ph] (2021); ÄB, S Wolf, PRSA 474 (2018); M Araújo, P Allard Guerin, ÄB, PRA 96 (2017); ÄB, S Wolf, NJP 18:1 (2016); ÄB, S Wolf, NJP 18:3 (2016); M Araújo *et al.,* Quantum 1 (2017); ÄB *et al.,* Class Quant Grav 36 (2019);

Main questions

What is the origin of causal order?

Are closed time-like curves mathematical artefacts of general relativity?

Are there "severe" consequence of violations of causal order?

Will there be closed time-like curves in quantum gravity?

Thank you for you attention.