Porting Quantum Research to Relativity

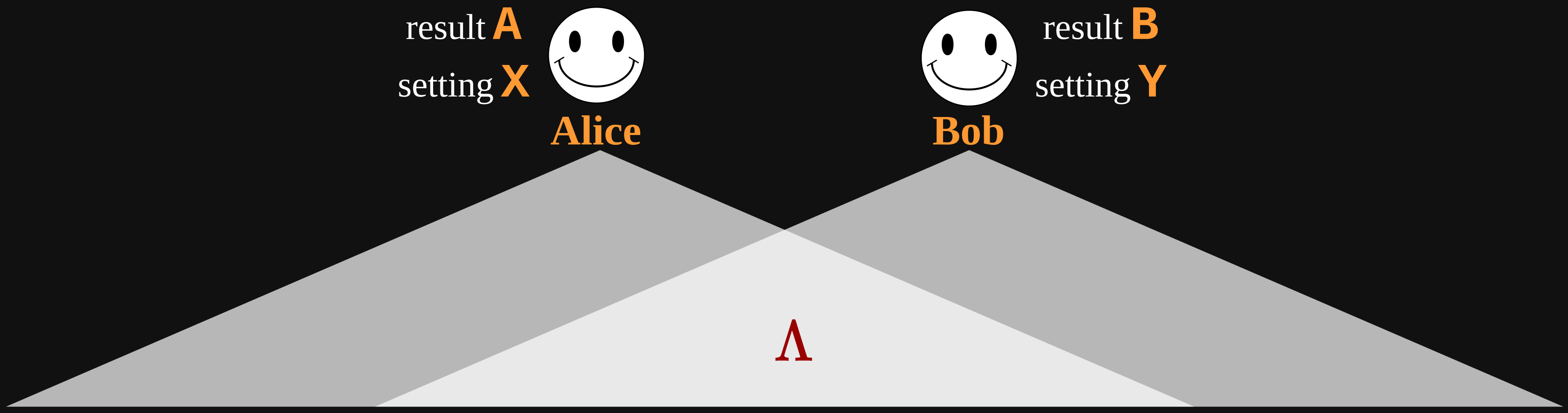
The Möbius Test

joint work with Eleftherios-Ermis Tselentis

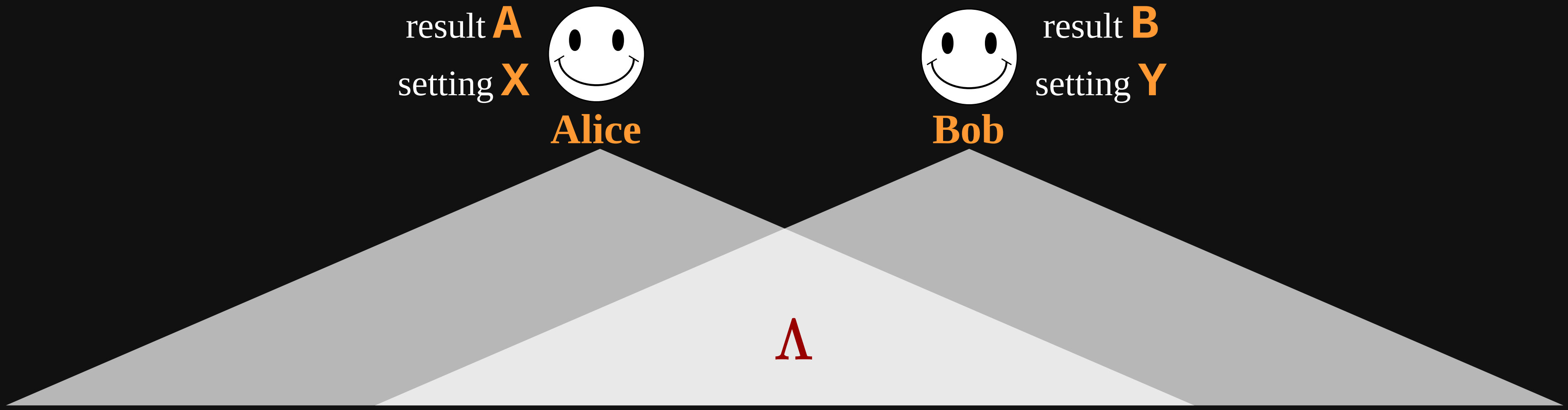
Ämin Baumeler, Università della Svizzera italiana

February 7, 2025 @ **CNP3** in Gandria

The Bell Test



$$p(a,b|x,y) = \sum_{\lambda} p(\lambda)\ \ p(a|x,\lambda)\ \ p(b|y,\lambda)$$



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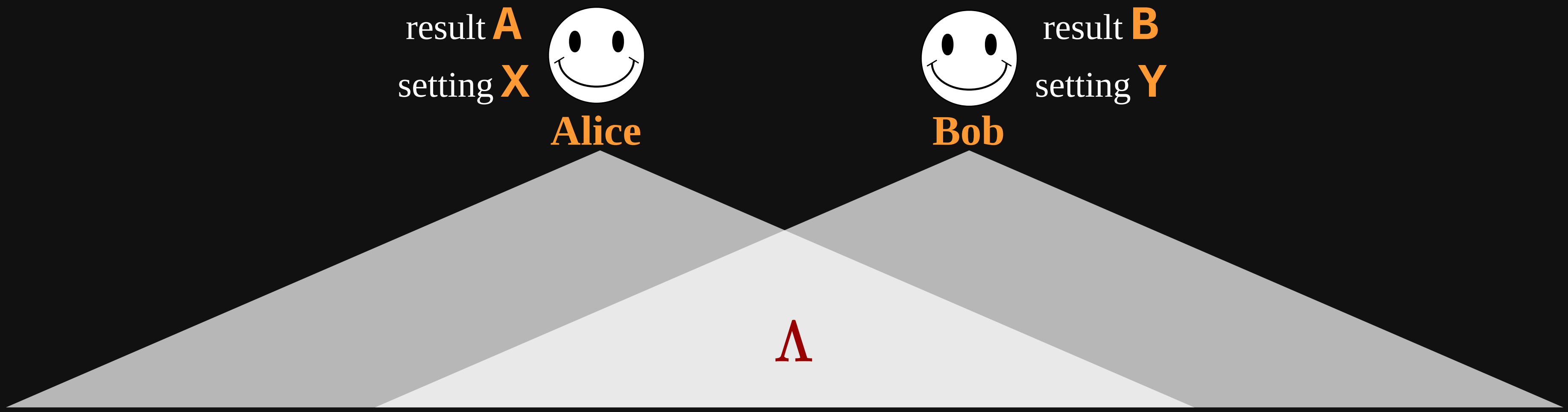
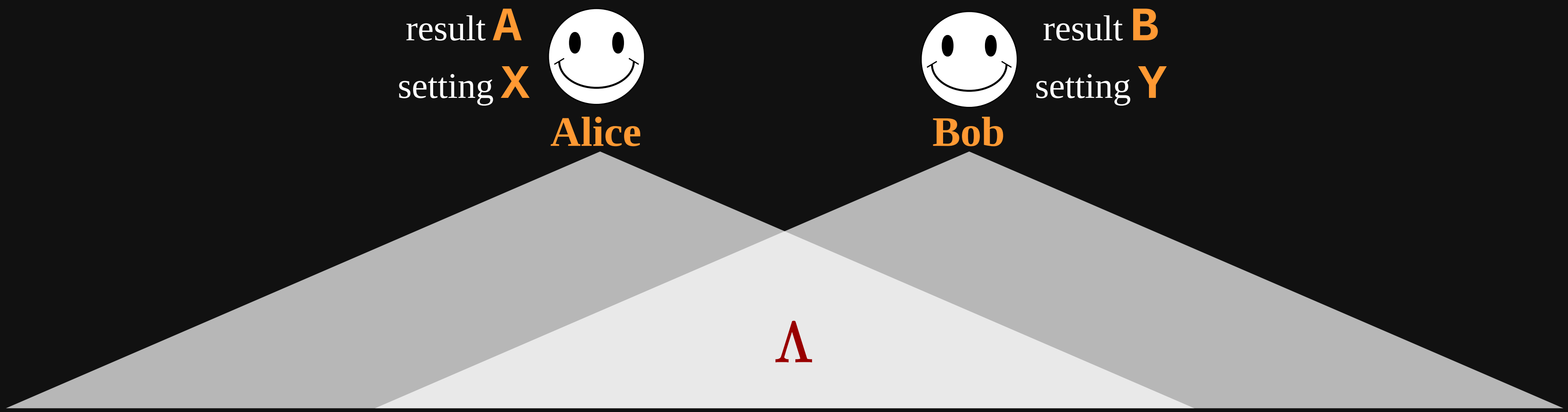
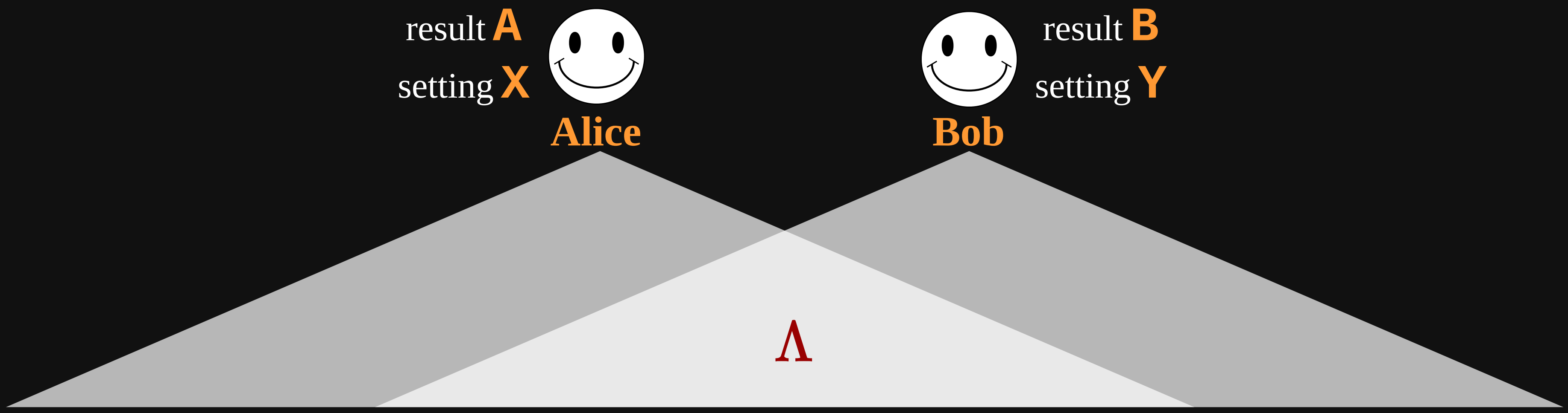


$$p(a,b|x,y) = \sum_\lambda p(\lambda)\; p(a|x,\lambda)\; p(b|y,\lambda)$$



$$p(a,b|x,y) = \sum_{\lambda} p(\lambda)\ \ p(a|x,\lambda)\ \ p(b|y,\lambda)$$



$$p(a,b|x,y) = \sum_{\lambda} p(\lambda)\ \ p(a|x,\lambda)\ \ p(b|y,\lambda)$$

$$\Pr[A \oplus B = X \cdot Y] \leq 3/4$$

The Bell Test: device independence

Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly nonlocal structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

II. Formulation

With the example advocated by Bohm and Aharonov [6], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet spin state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. If measurement of the component $\vec{\sigma}_1 \cdot \vec{a}$, where $\vec{a}$ is some unit vector, yields the value +1 then, according to quantum mechanics, measurement of $\vec{\sigma}_2 \cdot \vec{a}$ must yield the value −1 and vice versa. Now we make the hypothesis [2], and it seems one at least worth considering, that if the two measure-

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E P I S T E M O L O G I C A L L E T T E R S
L E T T R E S E P I S T E M O L O G I Q U E S
E P I S T E M O L O G I S C H E B R I E F E

- - -

Hidden Variables and Quantum Uncertainty
(Written Symposium, 9th Issue)

Variables cachées et indéterminisme quantique
(Symposium ocrit, 9ème livraison)

Verborgene Parameter und Quanten-Unbestimmtheit
(Schriftliches Symposium, 9.Heft)

march 1976 mars

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16.0	M.Mugur-Schächter	- What is at Stake in the experiments on Bell's Inequality?	1
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14.3	O.Costa de Beauregard	- Remarques à propos de 14.1 et 14.2	25
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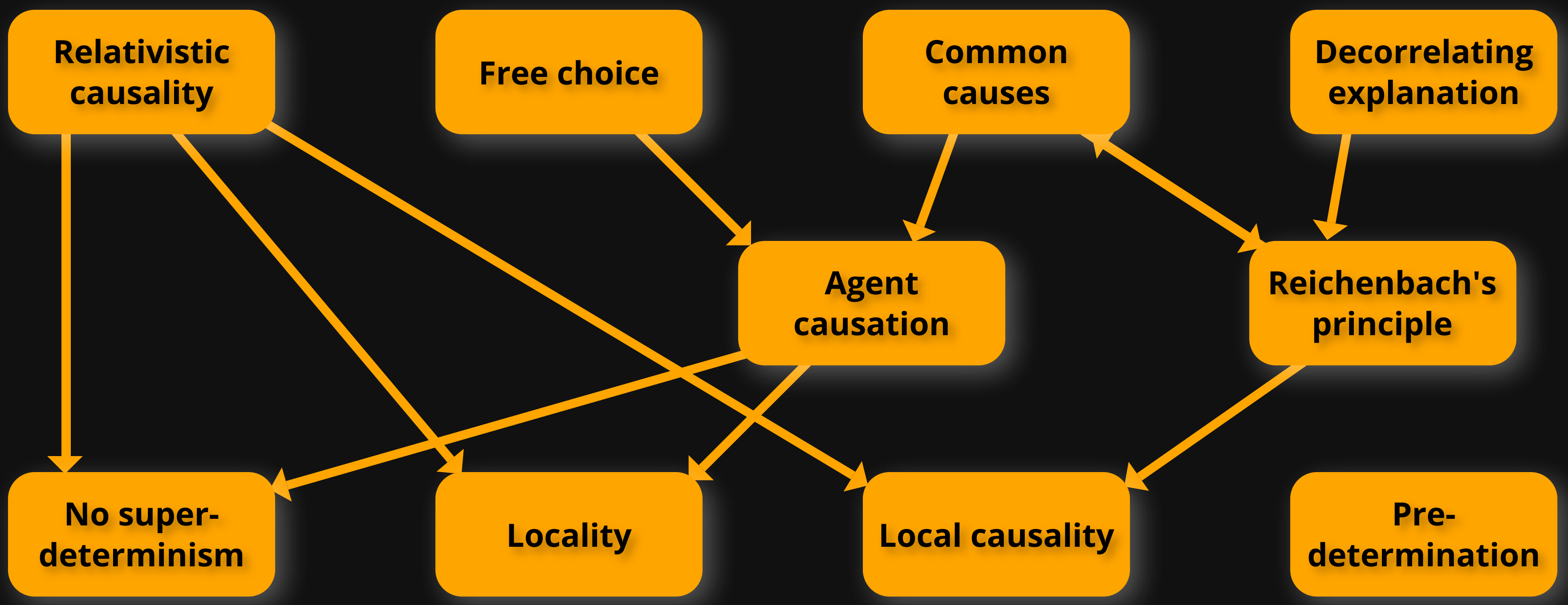
Postulates

Locality

Local causality

Pre-
determination

Web of postulates



"Quantum correlations falsify the hypothesis that, in any laboratory, nature carries the answer to any question which may be put there, and answers without knowing which questions are being put elsewhere."

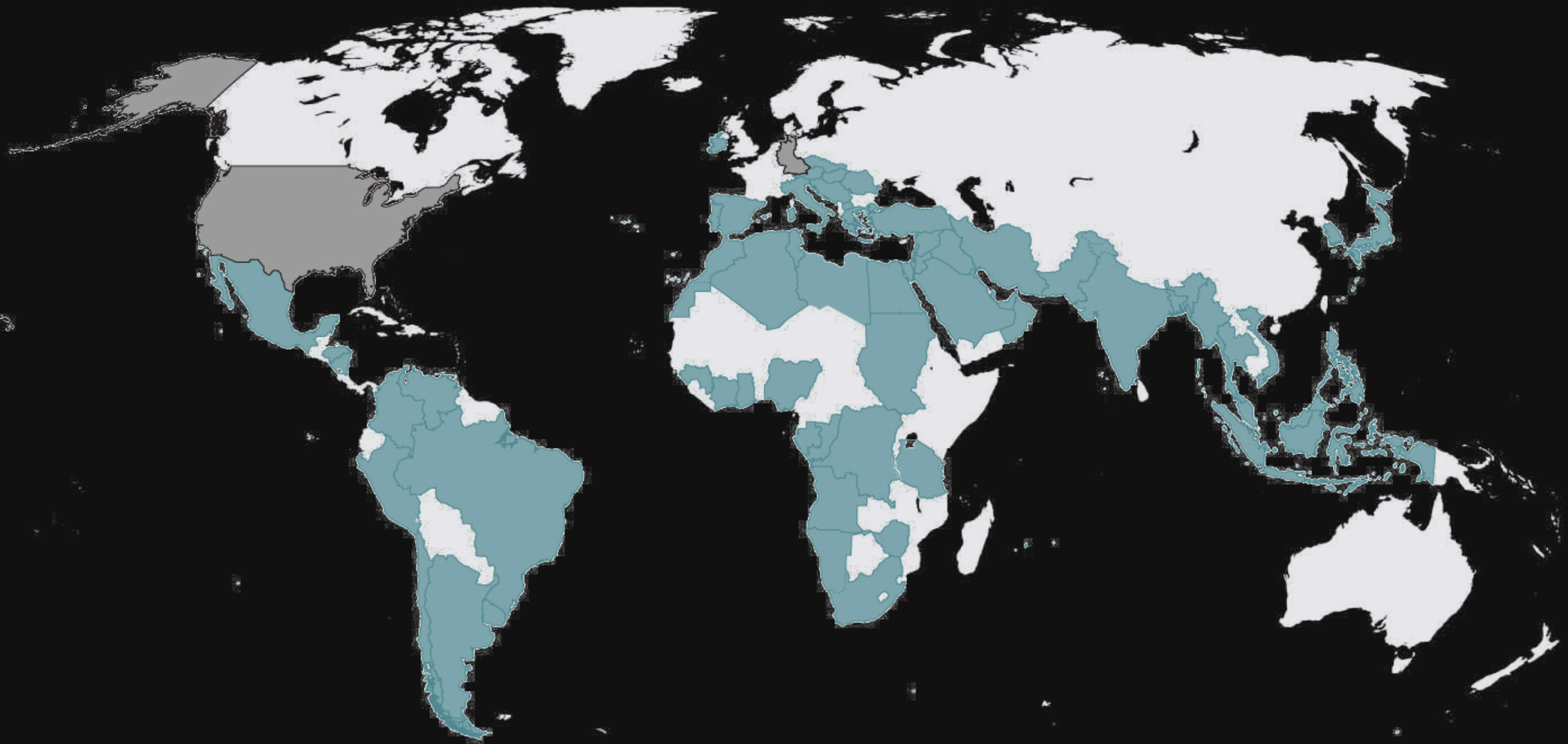
Spontaneous correlations:

$$p(a,b|x,y) \neq \sum_{\lambda} p(\lambda)\ \ p(a|x,\lambda)\ \ p(b|y,\lambda)$$

Independent of physical degrees of freedom, apparatus, physical systems, laws, ...

Applications: Cryptography

- Self testing
- Quantum key distribution
- Randomness amplification



The Bell Test: polytope theory

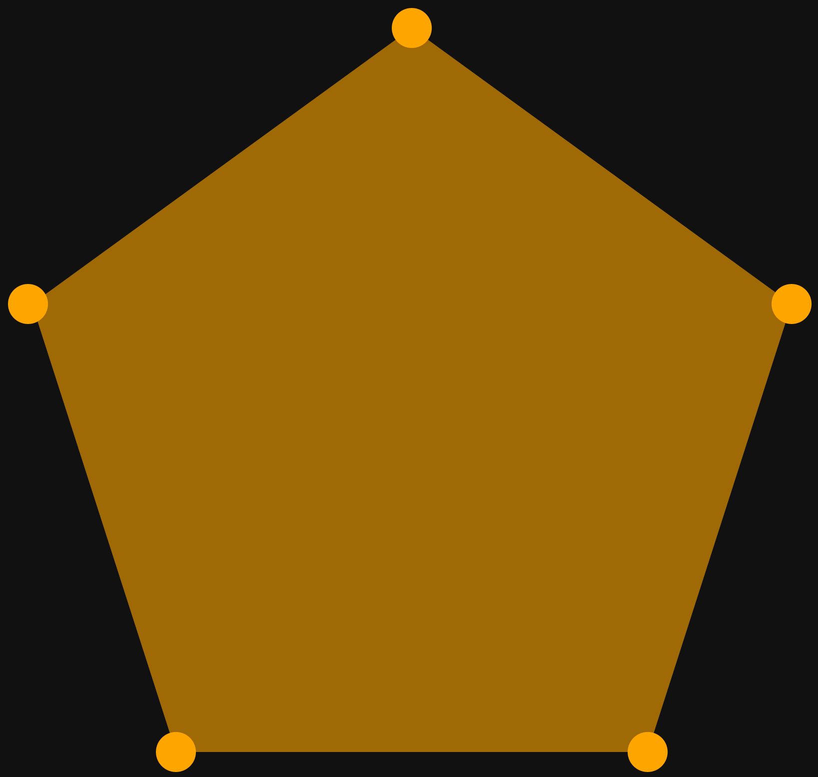
$$p(a, b|x, y) = \sum_{\lambda} p(\lambda) \; p(a|x, \lambda) \; p(b|y, \lambda)$$

$$
\begin{aligned}
\vec{p} = (\ & p(00|00), p(01|00), p(10|00), p(11|00), \\
& p(00|01), p(01|01), p(10|01), p(11|01), \\
& p(00|10), p(01|10), p(10|10), p(11|10), \\
& p(00|11), p(01|11), p(10|11), p(11|11)\ )
\end{aligned}
$$

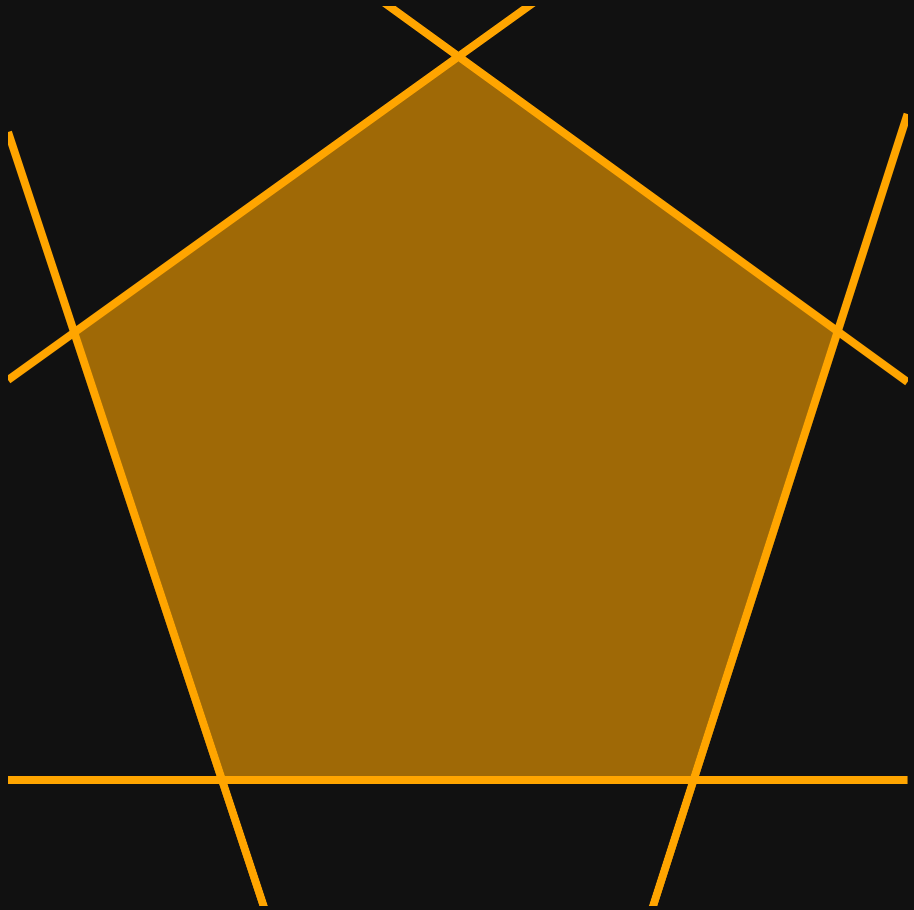
$$
\begin{aligned}
\vec{p} = (& \; p(00|00), p(01|00), p(10|00), \qquad\qquad , \\
& p(00|01), p(01|01), p(10|01), \qquad\qquad , \\
& p(00|10), p(01|10), p(10|10), \qquad\qquad , \\
& p(00|11), p(01|11), p(10|11), \qquad\qquad)
\end{aligned}
$$

$$
\begin{aligned}
\vec{p} = (\, & p(00|00), p(01|00), p(10|00), \\
& p(00|01), p(01|01), p(10|01), \\
& p(00|10), p(01|10), p(10|10), \\
& p(00|11), p(01|11), p(10|11)\,)
\end{aligned}
$$

Geometric representation: convex polytope



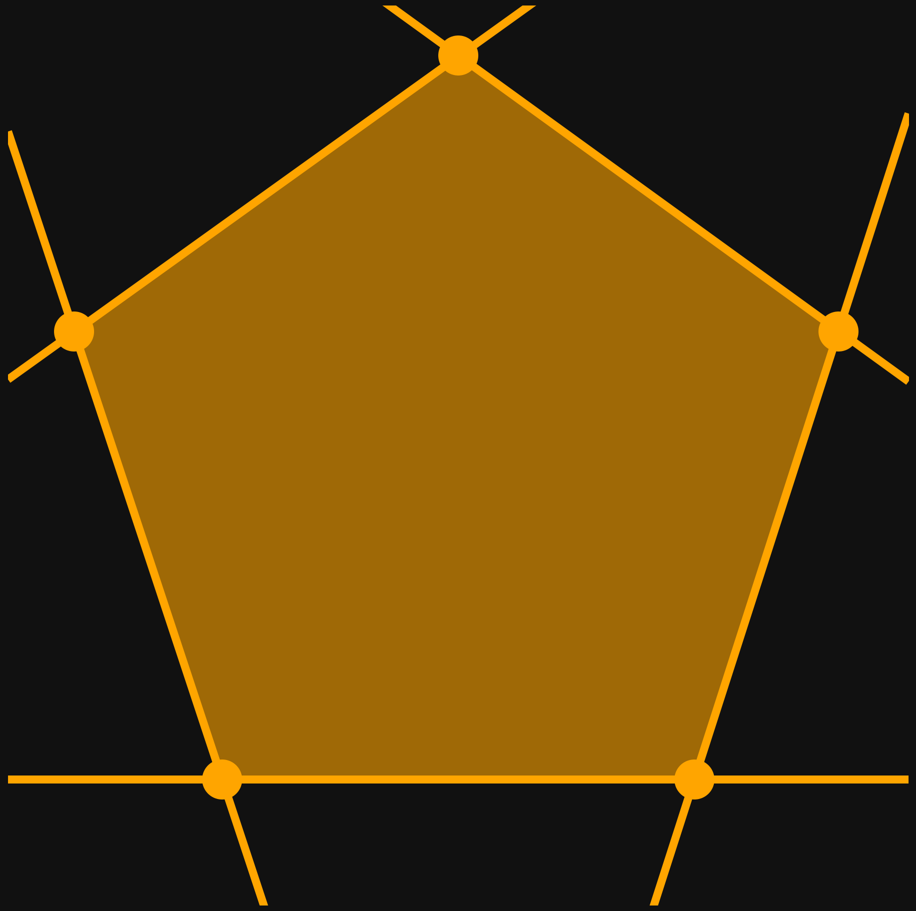
vertex representation



halfspace representation

$$p(a, b|x, y) = \sum_{\lambda} p(\lambda) \; p(a|x, \lambda) \; p(b|y, \lambda)$$

$$p(a, b|x, y) = [a = f_A(x, \lambda)]\ [b = f_B(y, \lambda)]$$

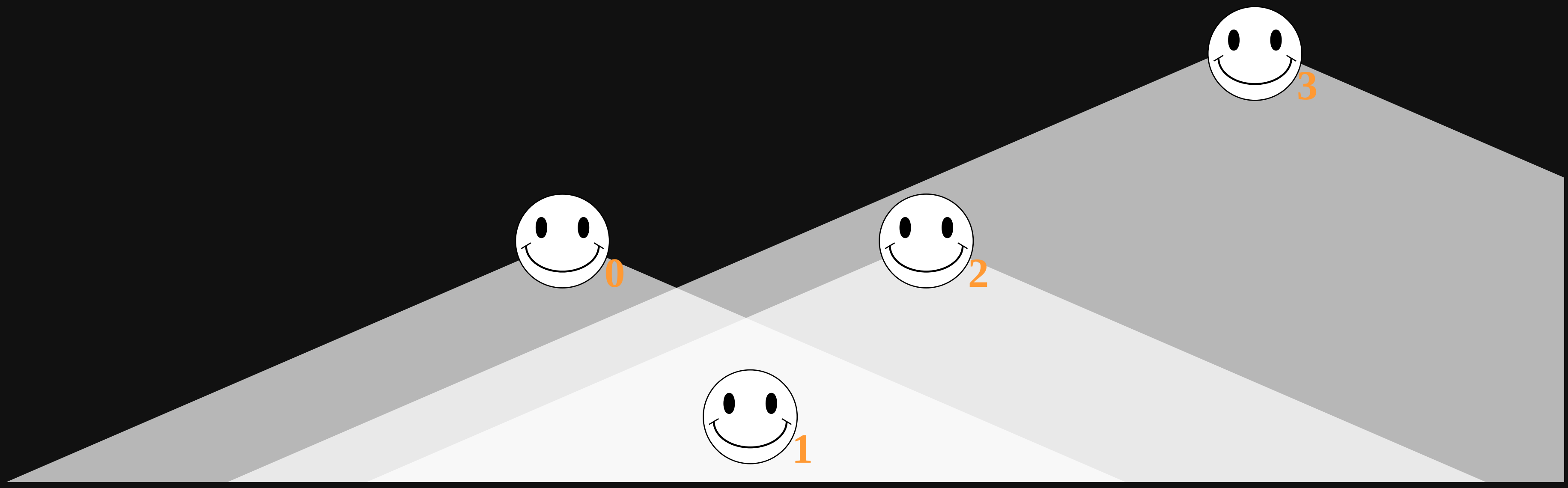


- Full dimensional
- Bell inequality is a **facet** *(11-dimensional)*

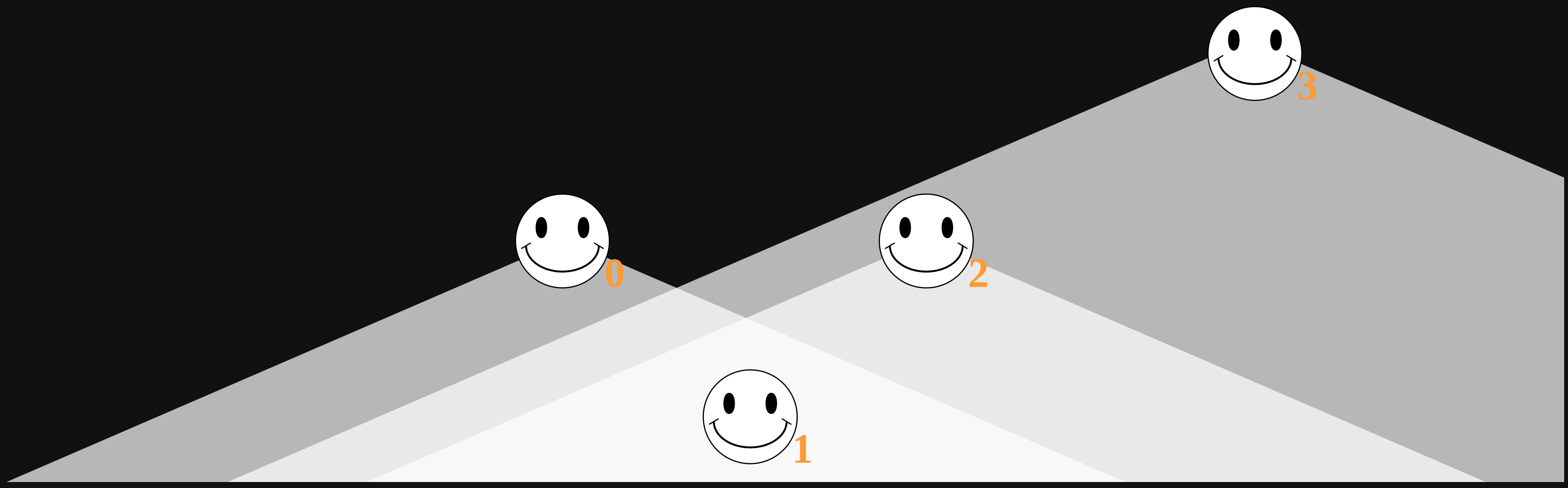
Outline

0. The Bell Test
1. **The Möbius Test**
2. Relativity

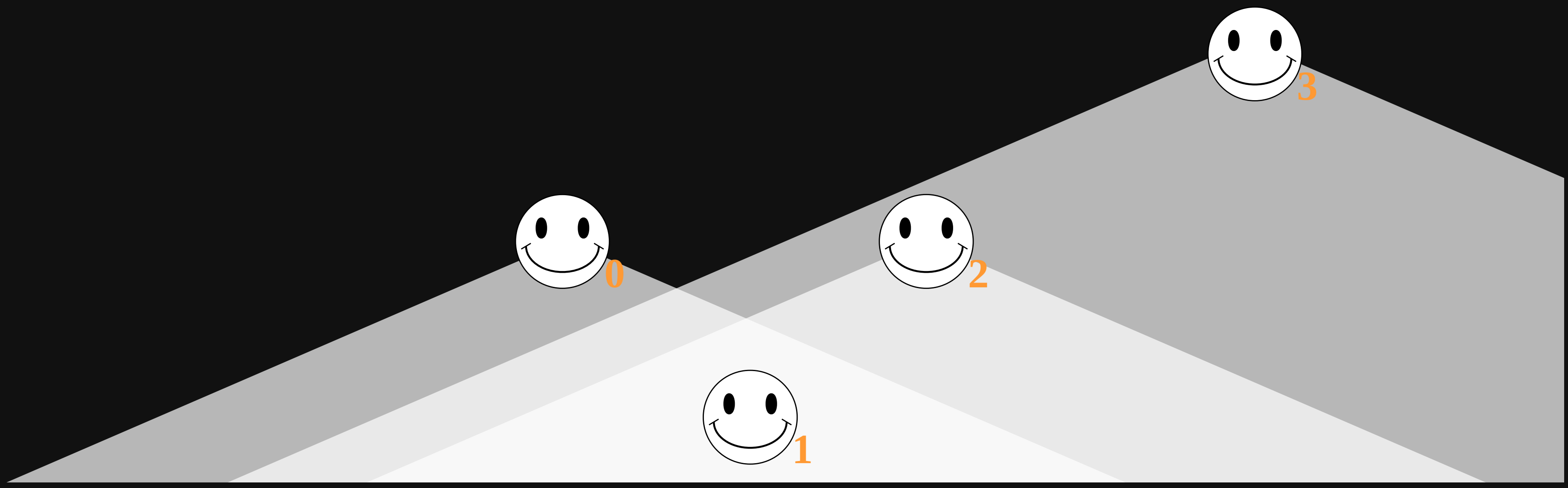
The Möbius Test



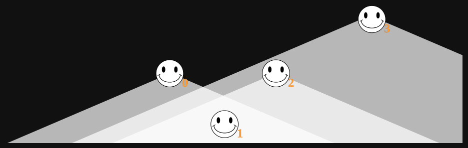
Observations at *fixed* spacetime coordinates



Observations obey *partial order*



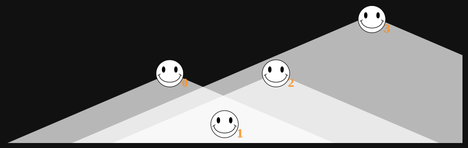
- Result A_3 may depend on setting X_1
- Result A_1 *cannot* depend on setting X_3



Decomposition of

$$p(a_0, a_1, a_2, \ldots | x_0, x_1, x_2, \ldots)$$

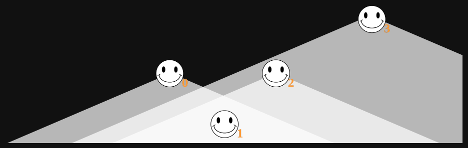
$$p(\underline{a}|\underline{x}) = \sum_\sigma p(\sigma) \prod_k p(a_k | \, \sigma, x_k, a_{\prec_\sigma k}, x_{\prec_\sigma k})$$



Decomposition of

$$p(a_0, a_1, a_2, \ldots \,|\, x_0, x_1, x_2, \ldots)$$

$$p(\underline{a}|\underline{x}) = \sum_\sigma p(\sigma) \prod_k p(a_k |\, \sigma, x_k, a_{\prec_\sigma k}, x_{\prec_\sigma k})$$



Decomposition of

$$p(a_0, a_1, a_2, \ldots | x_0, x_1, x_2, \ldots)$$

$$p(\underline{a}|\underline{x}) = \sum_\sigma p(\sigma) \prod_k p(a_k | \sigma, x_k, a_{\prec_\sigma k}, x_{\prec_\sigma k})$$

A simple scenario

$$p(\, a \mid s, r, x \,)$$

$$p(\, a \mid s, r, x \,)$$

- Two *distinct* party labels: s and r

$$p(\, a \mid s, r, x \,)$$

- Two *distinct* party labels: s and r
- A binary setting x by party s *(the "sender")*

$$p(\, a \mid s, r, x \,)$$

- Two *distinct* party labels: s and r
- A binary setting x by party s *(the "sender")*
- A binary result a by party r *(the "receiver")*

$$p(\ a \mid s, r, x\ )$$

$$= \sum_{\sigma : s \prec_\sigma r} p(\sigma) p(a|\sigma, s, r, x) + \sum_{\sigma : s \not\prec_\sigma r} p(\sigma) p(a|\sigma, s,$$

- Two *distinct* party labels: s and r
- A binary setting x by party s *(the "sender")*
- A binary result a by party r *(the "receiver")*

$$p(\, a \mid s, r, x \,)$$

$$= [s \prec_\sigma r][a = f(s, r, x)] + [s \not\prec_\sigma r][a = g(s, r)]$$

- Two *distinct* party labels: s and r
- A binary setting x by party s *(the "sender")*
- A binary result a by party r *(the "receiver")*

$$
\begin{aligned}
\vec{p} = (&\\
&p(0|A,B,0), p(1|A,B,0), p(0|A,B,1), p(1|A,B,1),\\
&p(0|A,C,0), p(1|A,C,0), p(0|A,C,1), p(1|A,C,1),\\
&p(0|B,A,0), p(1|B,A,0), p(0|B,A,1), p(1|B,A,1),\\
&p(0|B,C,0), p(1|B,C,0), p(0|B,C,1), p(1|B,C,1),\\
&p(0|C,A,0), p(1|C,A,0), p(0|C,A,1), p(1|C,A,1),\\
&p(0|C,B,0), p(1|C,B,0), p(0|C,B,1), p(1|C,B,1)\\
)&
\end{aligned}
$$

$$p(0|A, B, 0), \quad p(0|A, B, 1),$$
$$p(0|A, C, 0), \quad p(0|A, C, 1),$$
$$p(0|B, A, 0), \quad p(0|B, A, 1),$$
$$p(0|B, C, 0), \quad p(0|B, C, 1),$$
$$p(0|C, A, 0), \quad p(0|C, A, 1),$$
$$p(0|C, B, 0), \quad p(0|C, B, 1),$$

$$\left(p(0|A, B, 1), p(0|A, C, 1), p(0|B, A, 1), p(0|B, C, 1), p(0|C, A, 1), p(0|C, B, 1), \right)$$

$$
\begin{aligned}
&(\\
&p(0|A,B,0),p(0|A,B,1),\\
&p(0|A,C,0),p(0|A,C,1),\\
&p(0|B,A,0),p(0|B,A,1),\\
&p(0|B,C,0),p(0|B,C,1),\\
&p(0|C,A,0),p(0|C,A,1),\\
&p(0|C,B,0),p(0|C,B,1)\\
&)
\end{aligned}
$$

$$
\begin{gathered}
(\\
p(0|A,B,0), p(0|A,B,1),\\
p(0|A,C,0), p(0|A,C,1),\\
p(0|B,A,0), p(0|B,A,1),\\
p(0|B,C,0), p(0|B,C,1),\\
p(0|C,A,0), p(0|C,A,1),\\
p(0|C,B,0), p(0|C,B,1)\\
)
\end{gathered}
$$

$p(0|A,B,0) \neq p(1|A,B,1)$: signal from A to B

$$
\Big(
p(0|A,B,0) \oplus p(0|A,B,1),
p(0|A,C,0) \oplus p(0|A,C,1),
p(0|B,A,0) \oplus p(0|B,A,1),
p(0|B,C,0) \oplus p(0|B,C,1),
p(0|C,A,0) \oplus p(0|C,A,1),
p(0|C,B,0) \oplus p(0|C,B,1)
\Big)
$$

$$
\begin{aligned}
&(\\
&[A \to B] \in \{0,1\},\\
&[A \to C] \in \{0,1\},\\
&[B \to A] \in \{0,1\},\\
&[B \to C] \in \{0,1\},\\
&[C \to A] \in \{0,1\},\\
&[C \to B] \in \{0,1\},\\
&)
\end{aligned}
$$

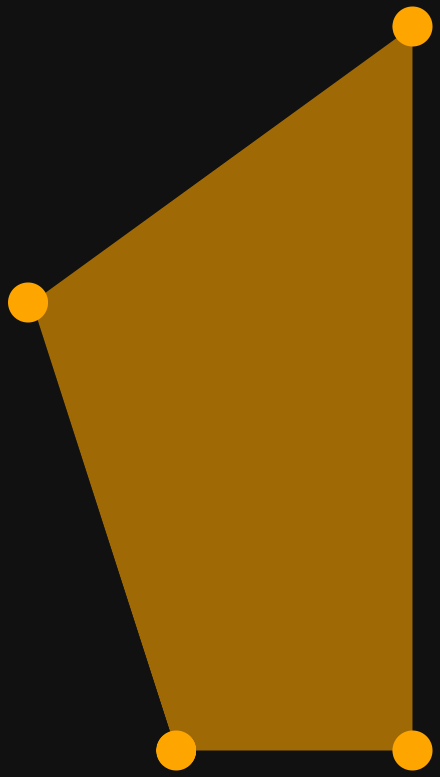
$$
\begin{gathered}
(\\
[A \to B],\\
[A \to C],\\
[B \to A],\\
[B \to C],\\
[C \to A],\\
[C \to B],\\
)
\end{gathered}
$$

Adjacency vector of a directed graph.

$$
\begin{array}{c}
(\\
[A \to B], \\
[A \to C], \\
[B \to A], \\
[B \to C], \\
[C \to A], \\
[C \to B], \\
)
\end{array}
$$

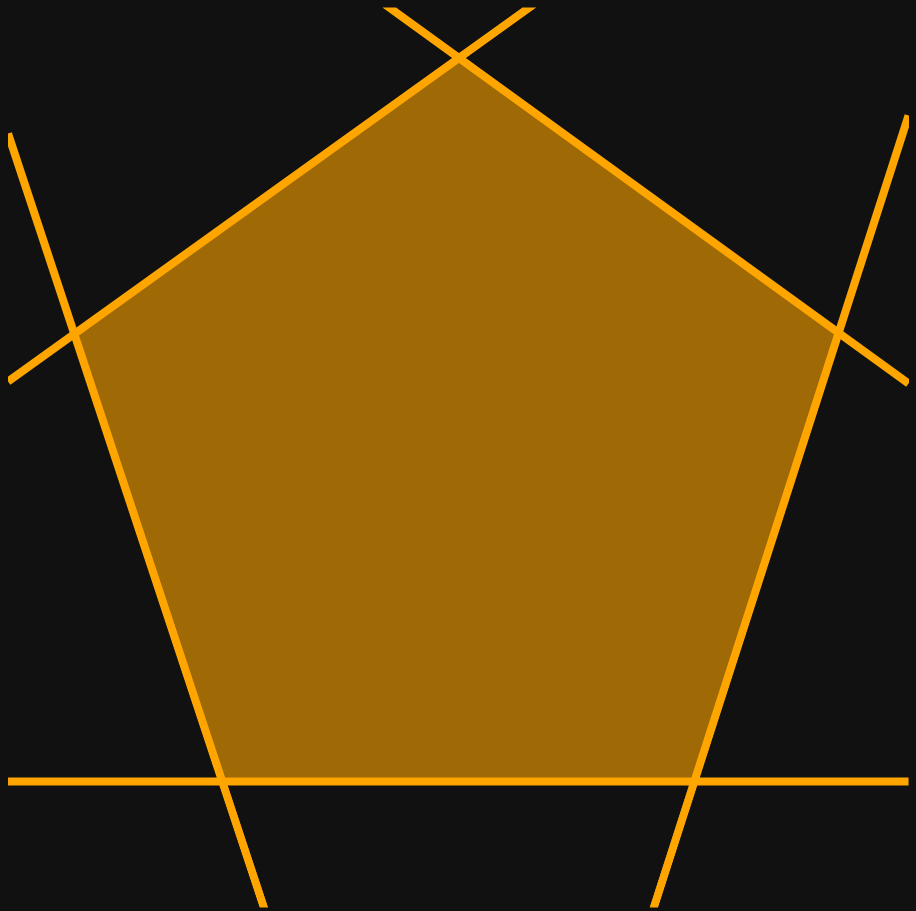
Adjacency vector of a directed acyclic graph.

- Projection: $2n(n-1)$ dim. to $n(n-1)$ dim.



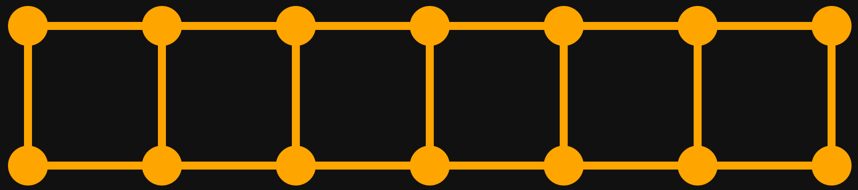
- Projection: $2n(n-1)$ dim. to $n(n-1)$ dim.
- Extremal points correspond to DAGs

- Projection: $2n(n-1)$ dim. to $n(n-1)$ dim.
- Extremal points correspond to DAGs
- Some halfspaces known since 1985

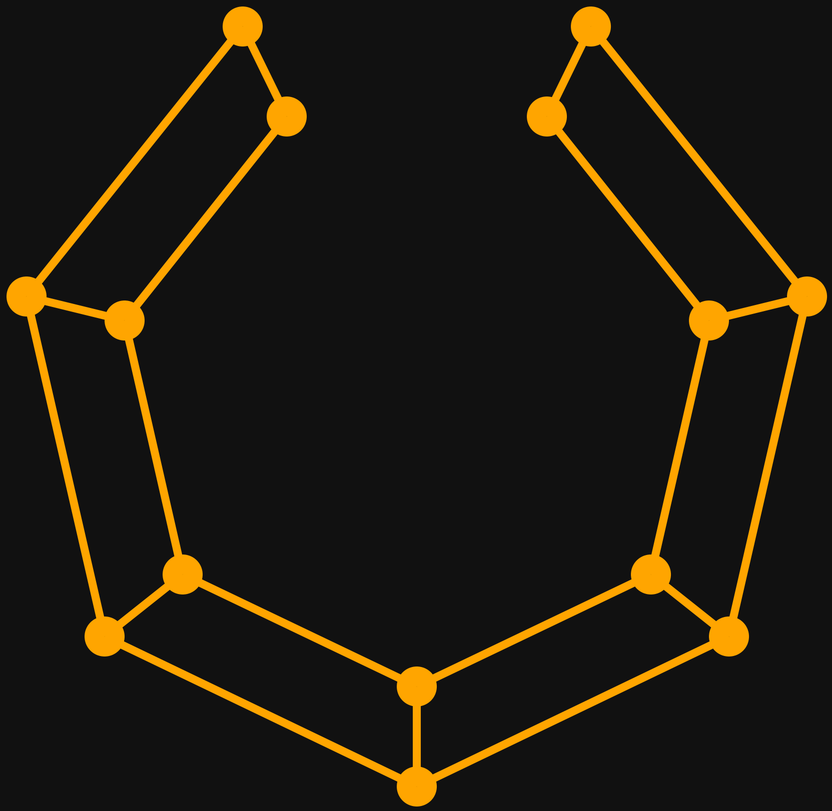


- Projection: $2n(n-1)$ dim. to $n(n-1)$ dim.
- Extremal points correspond to DAGs
- Some halfspaces known since 1985
- Lifting theorem

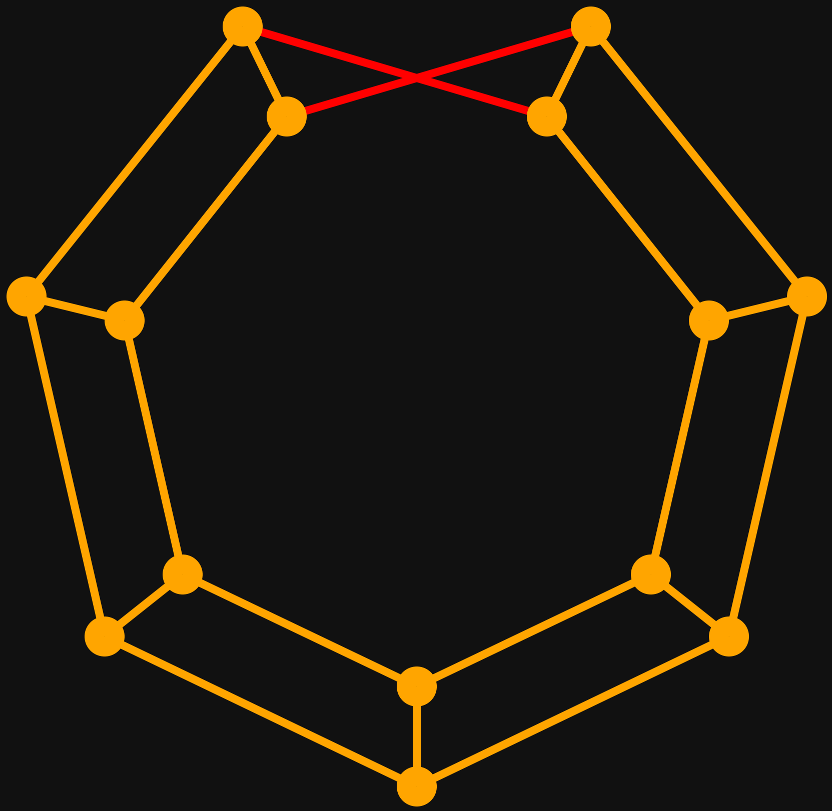
The Möbius graph



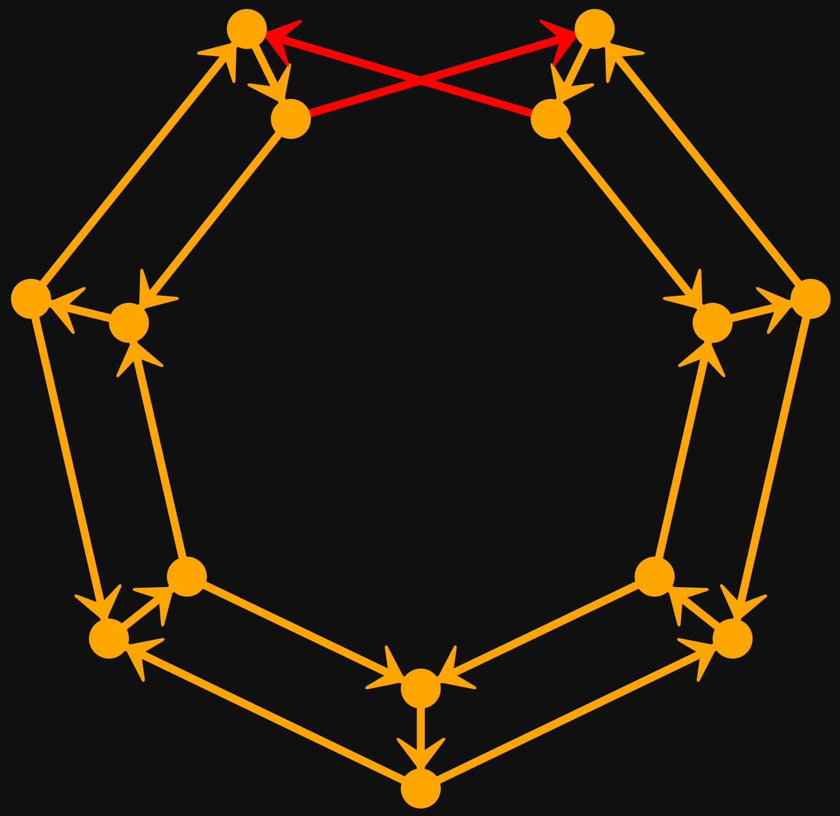
The Möbius graph



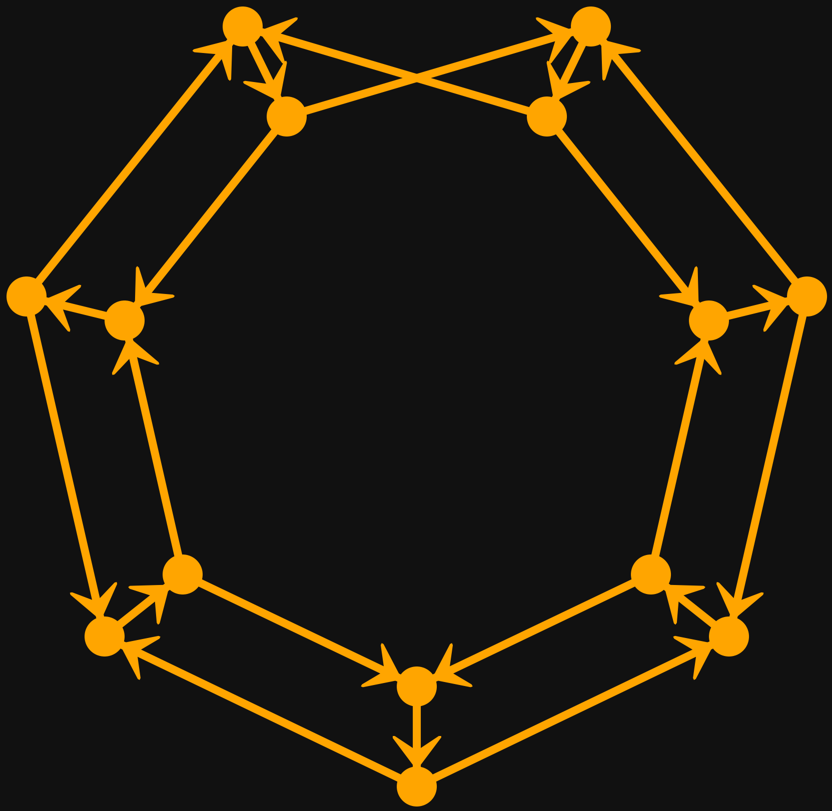
The Möbius graph



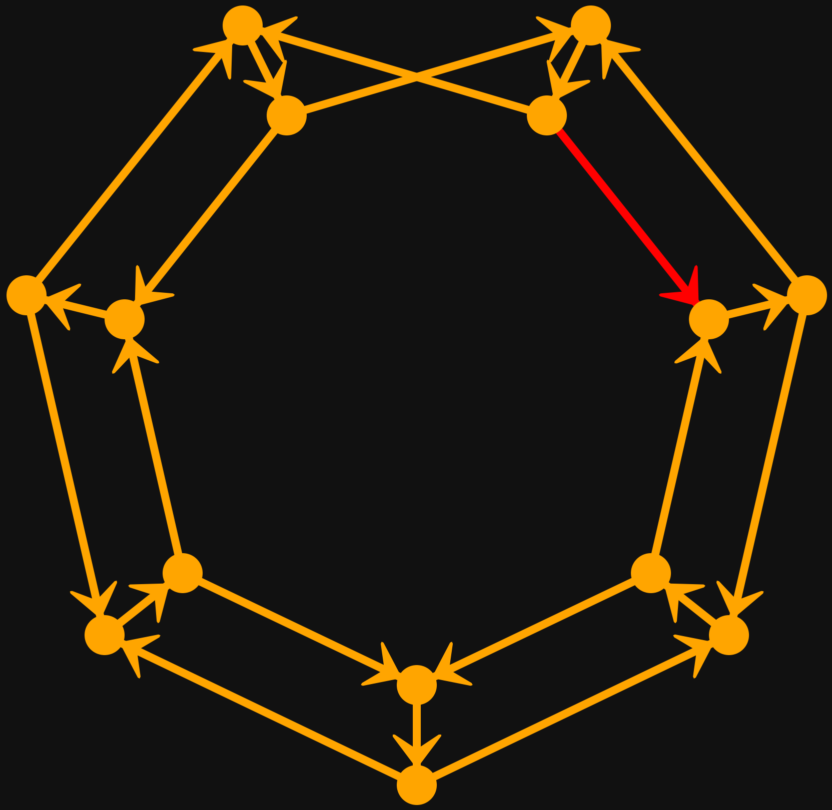
The Möbius graph



The Möbius inequality

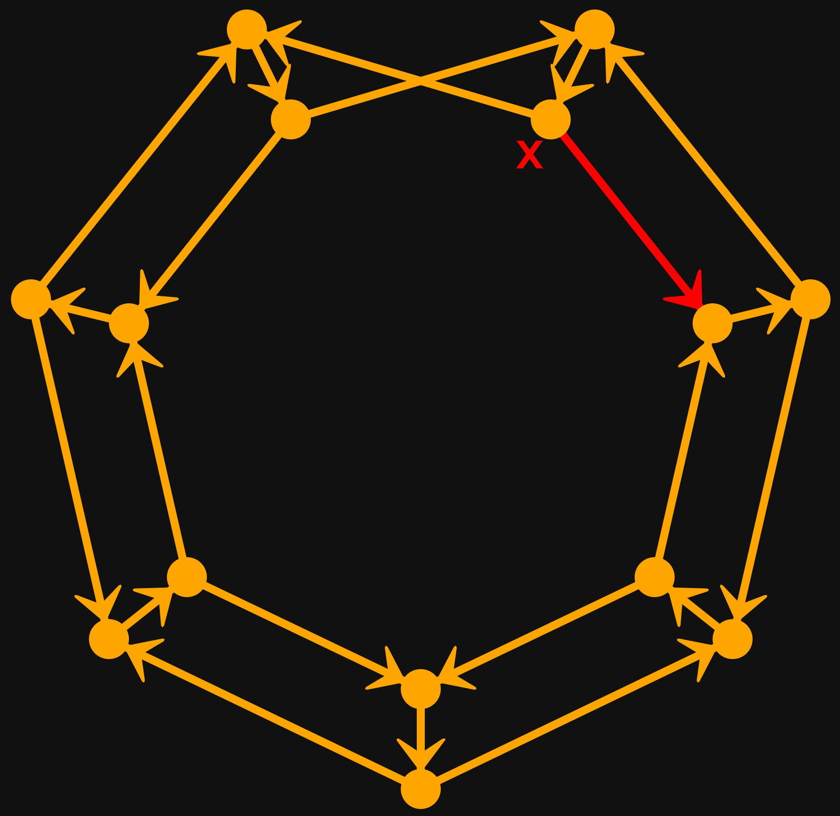


The Möbius inequality



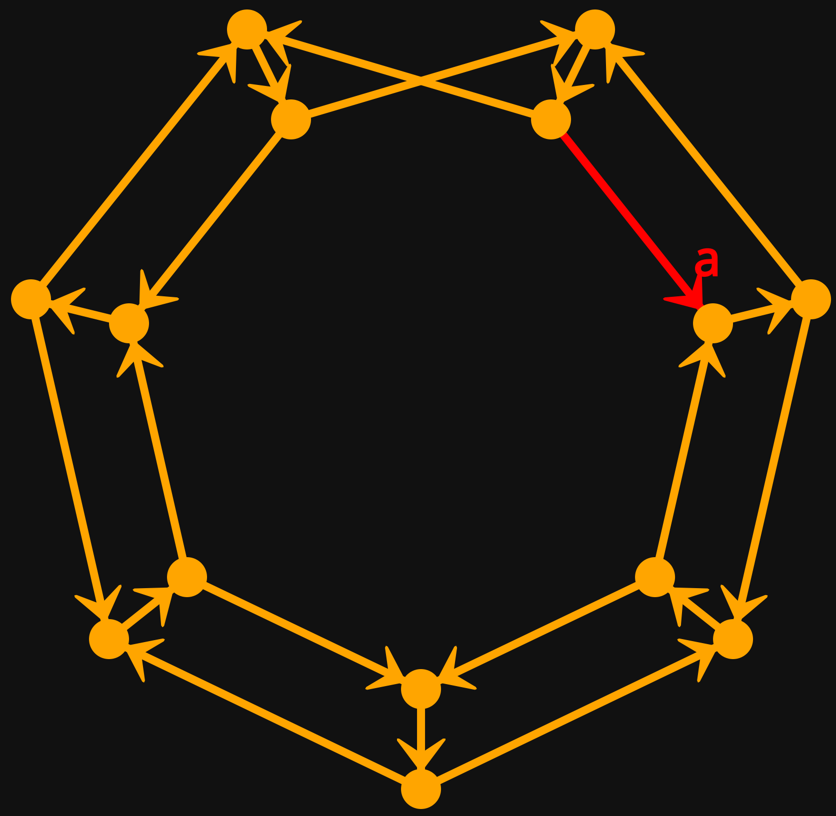
Referee announces random arc $s \longrightarrow r$

The Möbius inequality



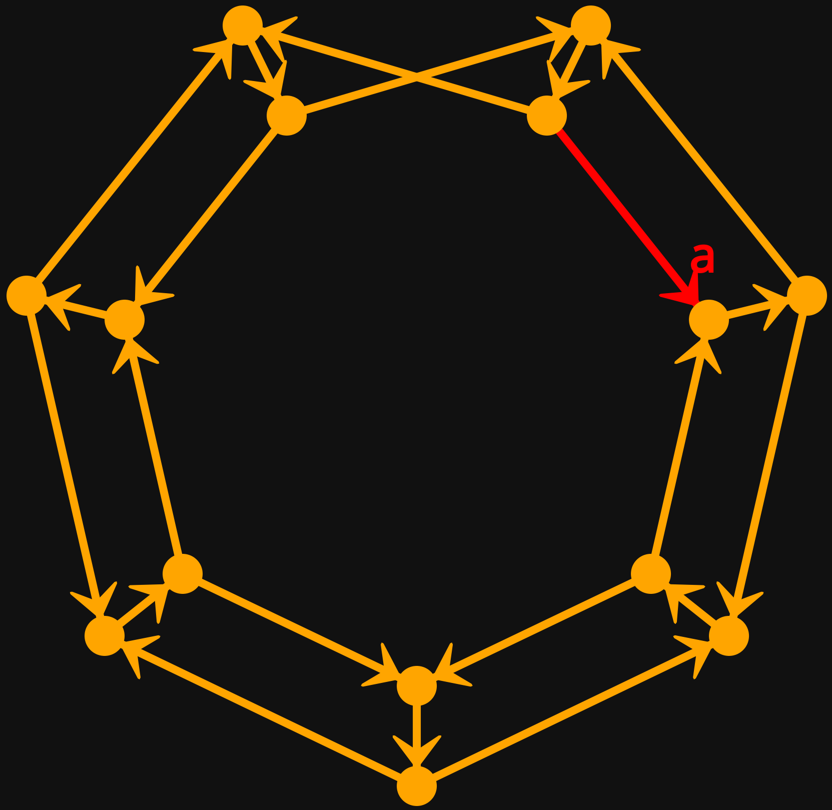
... and announces random bit x to the "sender" s

The Möbius inequality



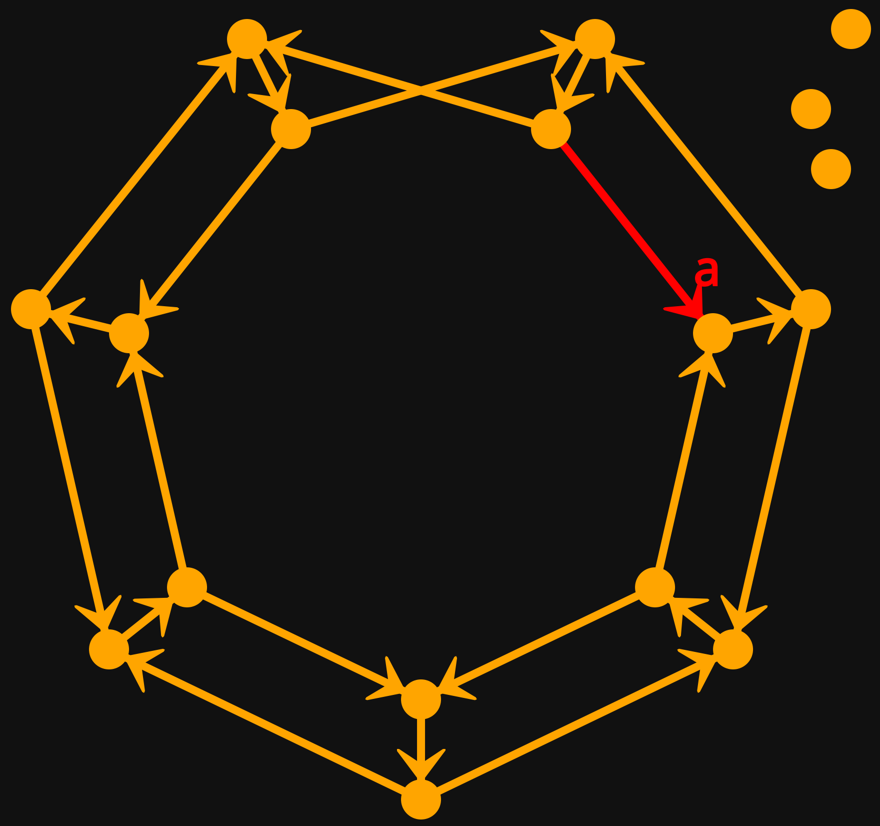
The parties **win** if the "receiver" r outputs $a = x$

The Möbius inequality



$$\Pr[A = X] \leq 1 - \frac{k+1}{12k} \leq \frac{11}{12}$$

The Möbius inequality



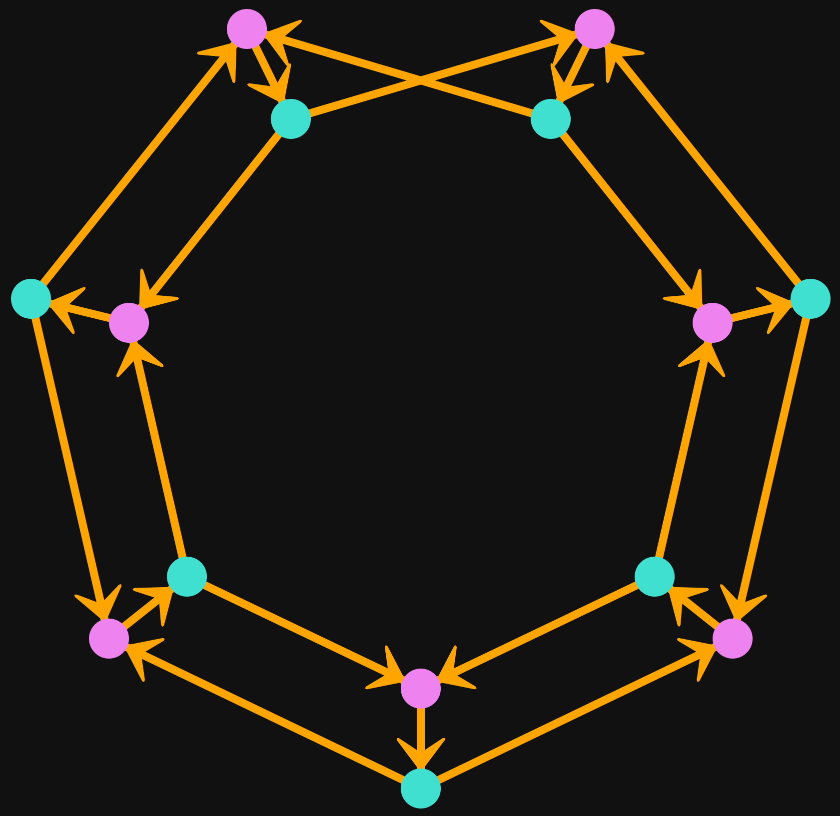
$$\Pr[A = X] \leq 1 - \frac{k+1}{12k} \leq \frac{11}{12}$$

Violation of the Möbius inequality …

… proves *incompatibility* with partial order.

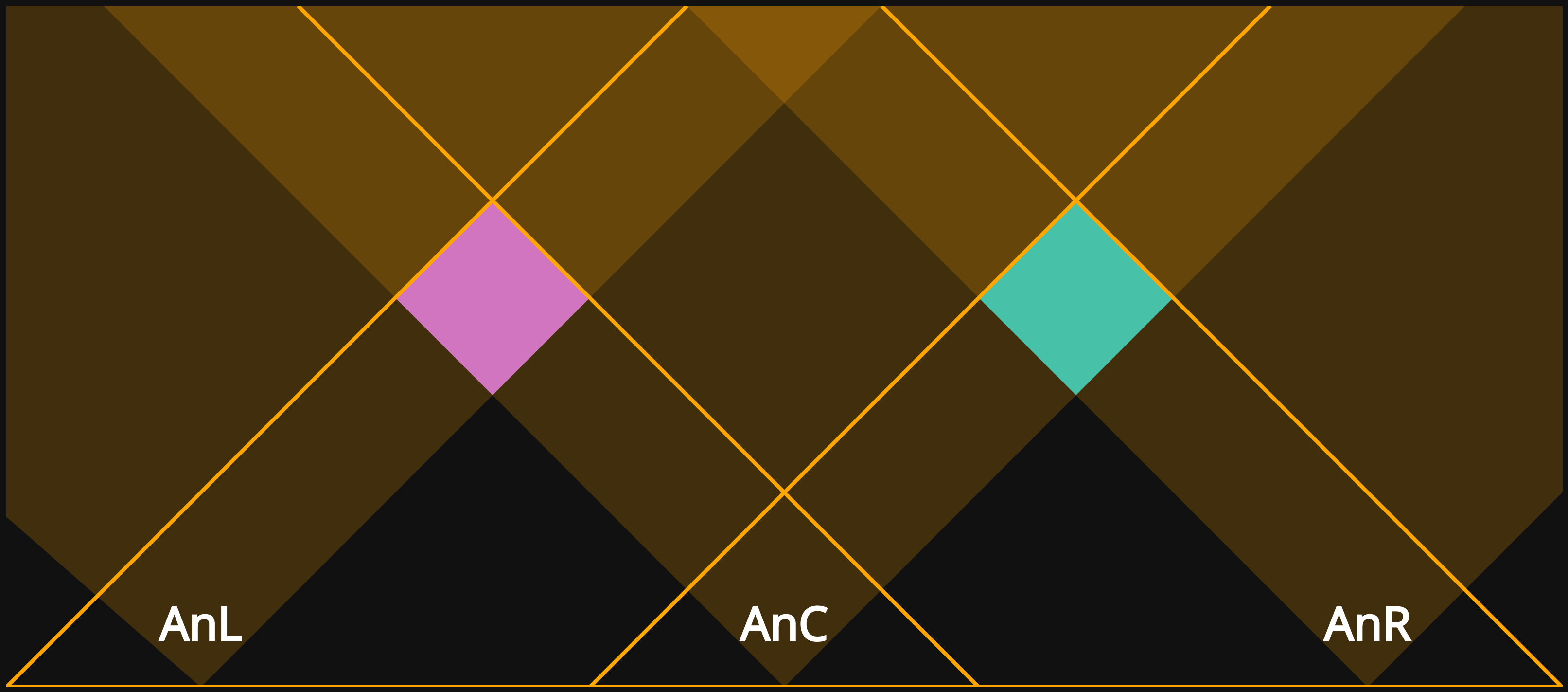
Relativity

The Möbius graph



Special relativity

- Non-overlapping **causal diamonds**: No violation



- AnC: $s \rightarrow r$, bit x_C
- AnL: x_L; AncR: x_R
- $s \in S_L : x := x_L \oplus x_C$
- $s \in S_R : x := x_R \oplus x_C$
- Collectors ensure that a is produced timely

General relativity

In principle a party in the common past of both diamonds can manipulate the spacetime structure of the future lightcone: **violation.**

Thank You

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Porting Quantum Research to Relativity

The Möbius Test

joint work with Eleftherios-Ermis Tselentis
Ämin Baumeler, Universitá della Svizzera italiana
February 7, 2025 @ **CNP3** in Gandria