FIXED POINTS OF QUANTUM EVOLUTION ON INDEFINITE CAUSAL STRUCTURES

Causality in the Quantum World
Anacapri, September 2019

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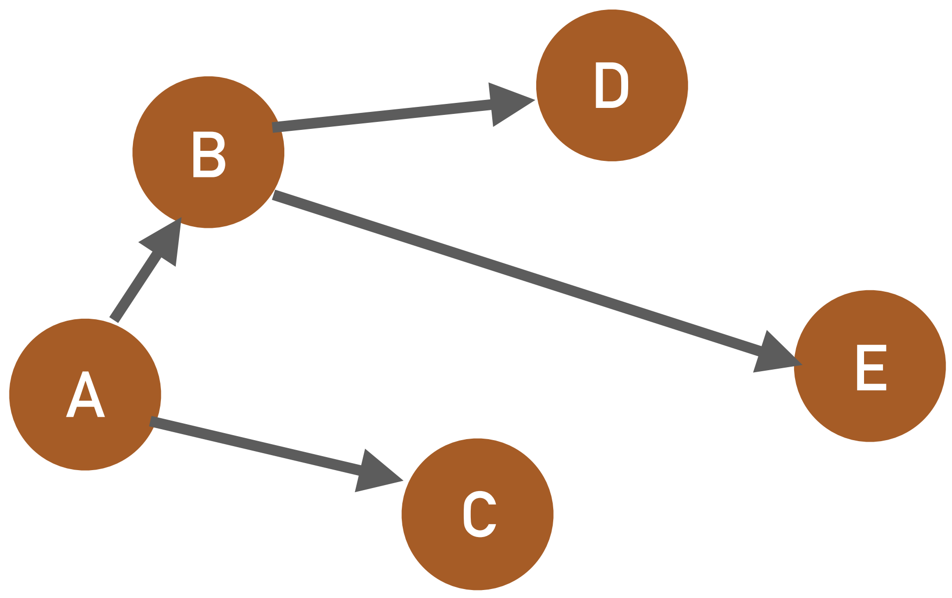
OUTLINE

- **Prelude:**
  On causality and the process-matrix framework
- **Motivation:**
  What we want to do and why: Find fixed points
- **Intermezzo and Intermezzo^2:**
  The classical case and computational complexity
- **Preliminary results:**
  Recursive quantum fixed points
- **Finale:**
  Challenges

ON CAUSALITY

- Cause-effect relations



- Traditional assumptions
  a) No cycles

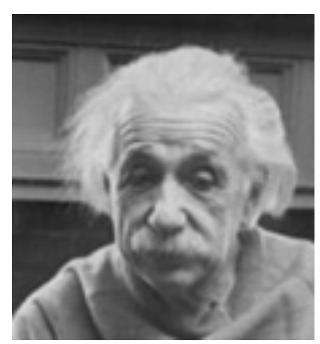
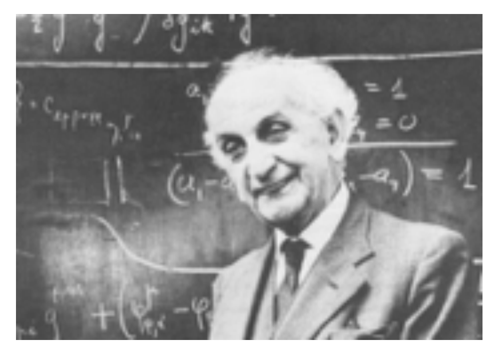
  b) „fixed“

MOTIVATIONS TO RELAX THESE ASSUMPTIONS

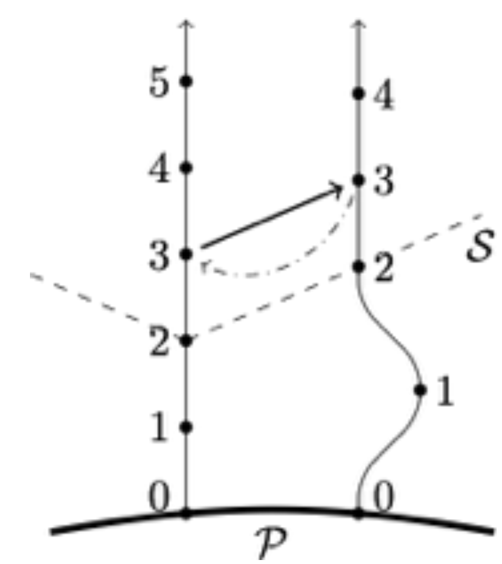
- Technical interest;
  Why not?!

- Cultural-philosophical reasons
  Parmenides, *etc.*

- General relativity
  Einstein, Lanczos, Gödel, Thorne, *etc.*


- Quantum theory
  (superposition principle)



- Overcome conceptual challenges of quantum theory?
  E.g., Parisian zig-zag model

THE PROCESS-MATRIX FRAMEWORK


- <u>Assumptions</u>

  i) Isolated parties with single interaction

  ii) For every choice of *quantum instruments* S_1, S_2, probabilities $P(x_1,x_2|a_1,a_2)$ well defined.

  iii) Probabilities are *linear* in the choice of *instruments*.

O. Oreshkov, F. Costa, Č. Brukner, *Nat. Commun.* 3, 1092 (2012).

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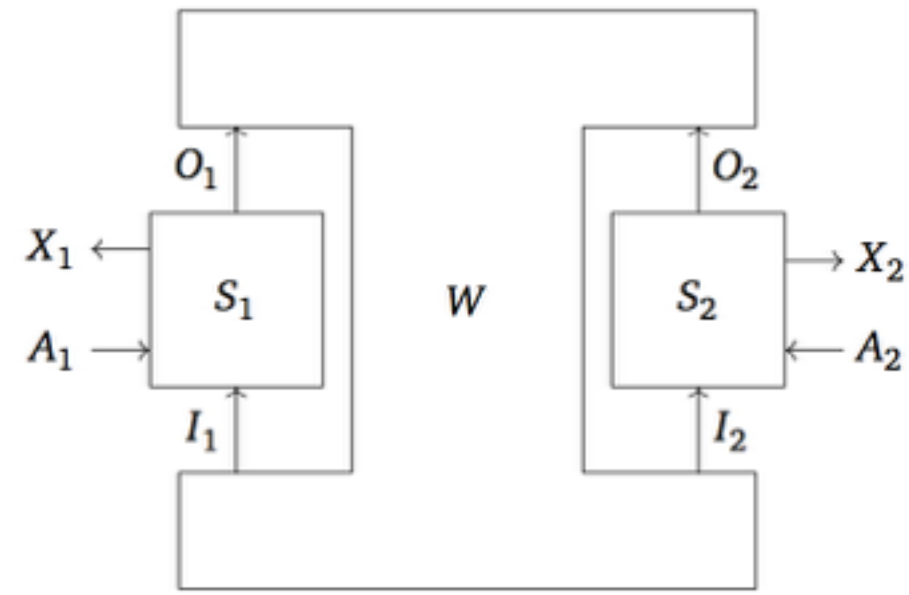

$$P(x_1, x_2|a_1, a_2) = \mathrm{Tr}\left((S_1^{x_1,a_1} \otimes S_2^{x_2,a_2})W\right)$$

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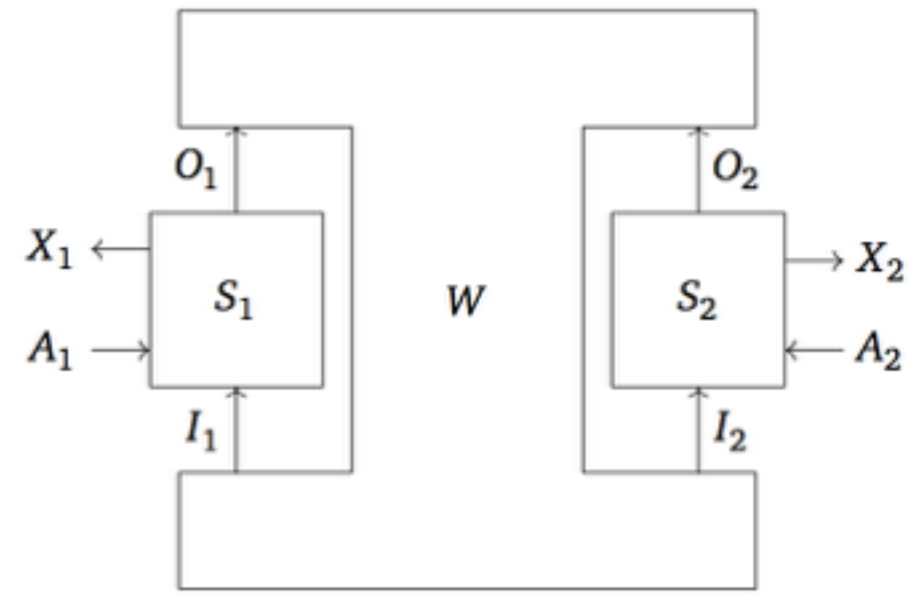


$$P(x_1, x_2|a_1, a_2) = \mathrm{Tr}\left((S_1^{x_1,a_1} \otimes S_2^{x_2,a_2})W\right)$$

The process matrix is a quantum channel!

O. Oreshkov, F. Costa, Č. Brukner, *Nat. Commun.* 3, 1092 (2012).

CAUSAL INEQUALITIES

- Device independent

- Describe the facets of the correlations obtainable in a *causal* way.
- Example: $\frac{1}{2}\left[P(x=b\,|\,b'=0)+P(y=a\,|\,b'=1)\right]\leq 3/4$

- The process-matrix framework *allows* for violations of such inequalities!

- Gretchenfrage: Can we realize such violations?!
  Ognyan: with space non-local variables.

O. Oreshkov, F. Costa, Č. Brukner, *Nat. Commun.* 3, 1092 (2012).

CAUSAL INEQUALITIES

- Gretchenfrage: Can we realize violations?!

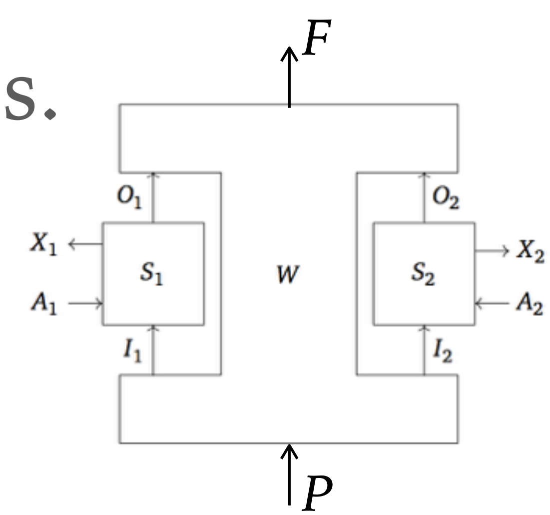
- Are they „just“ a mathematical artifact?

  (similar to the Gretchenfrage on the existence/realizability of closed time-like curves)

- If it’s a mathematical artifact, we better find reasons for that!

- One approach (that failed yet remains actual):
  Restrict the framework to *purifiable* process matrices.

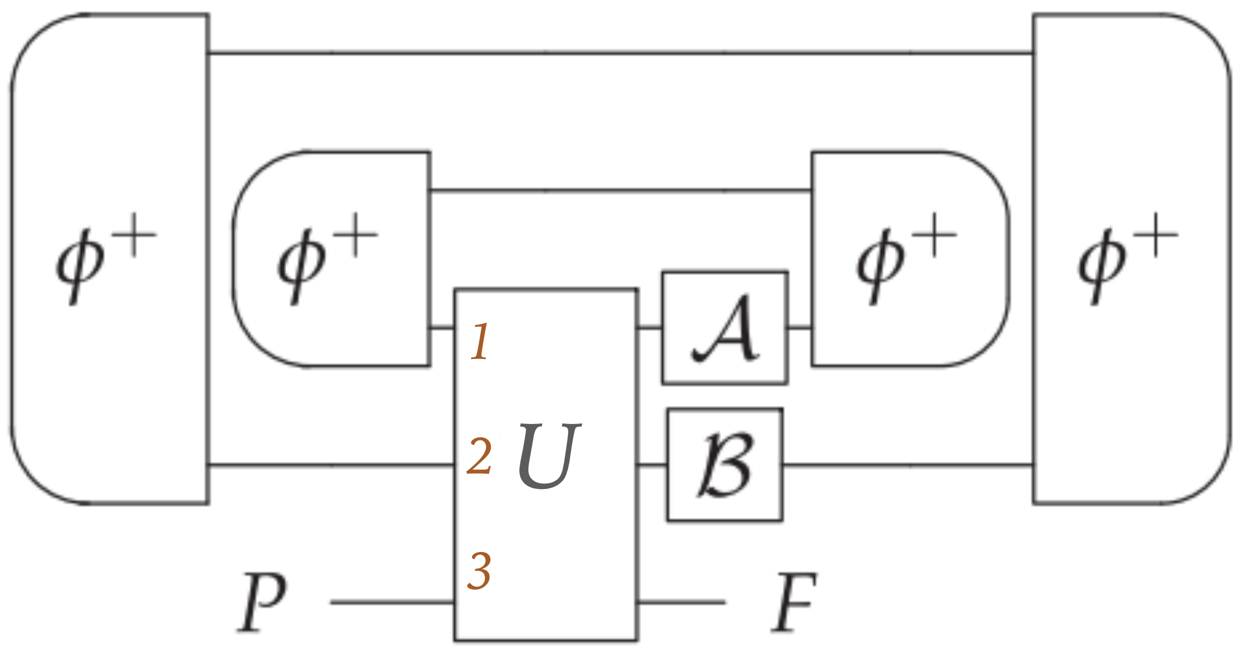
  Implication: Necessity of „source“ P and „sink“ F



M. Araújo, A. Feix, M. Navascués, Č. Brukner, Quantum 1, 10 (2017).

CONNECTION TO MATEUS' TALK

- Equivalence: Process-matrix framework and *linear* postselected closed time-like curves:

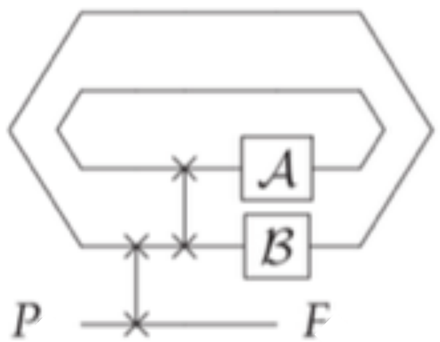


- Induced operations from *P* to *F*:
  P-CTC: $\mathrm{Tr}_{1,2}[(A\otimes B\otimes I)U]/z$ (fragile)
  Process matrices: $\forall A,B$ unitary: $\mathrm{Tr}_{1,2}[(A\otimes B\otimes I)U]$ unitary

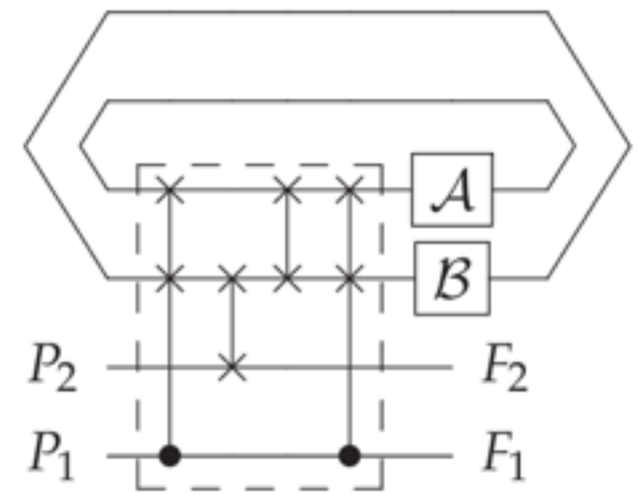
M. Araújo, P. Allard Guérin, ÄB, PRA 96, 52315 (2017).

EXAMPLES

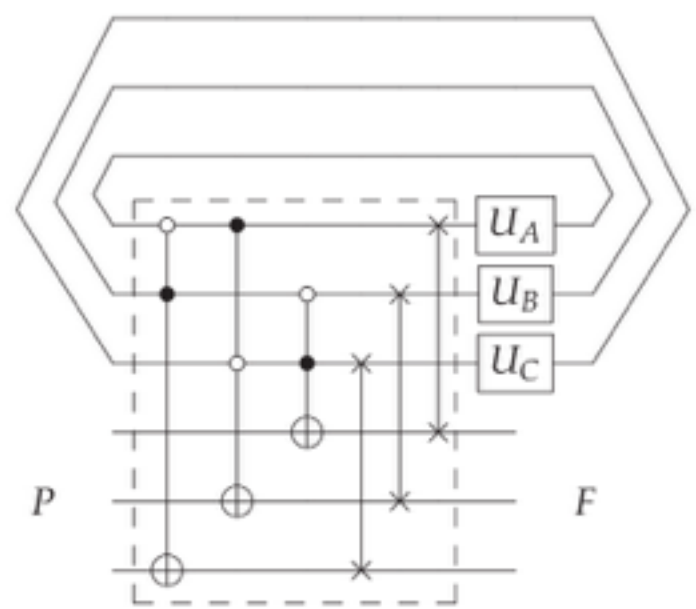
Alice before Bob: *BA*



Quantum switch



Violation of causal inequality

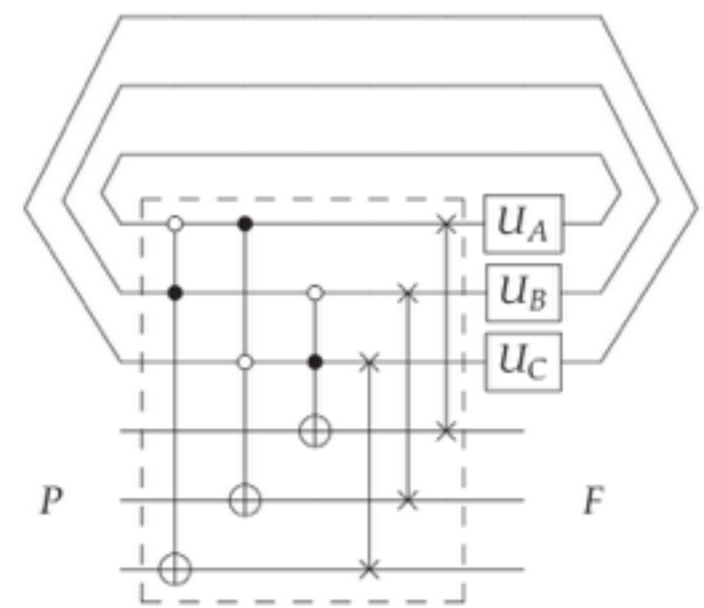


M. Araújo, P. Allard Guérin, ÄB, PRA 96, 52315 (2017).

MAIN QUESTION OF THIS TALK

- Can we talk about the quantum states *within*?
  General believe in P-CTC and two-state-formalism community: no.

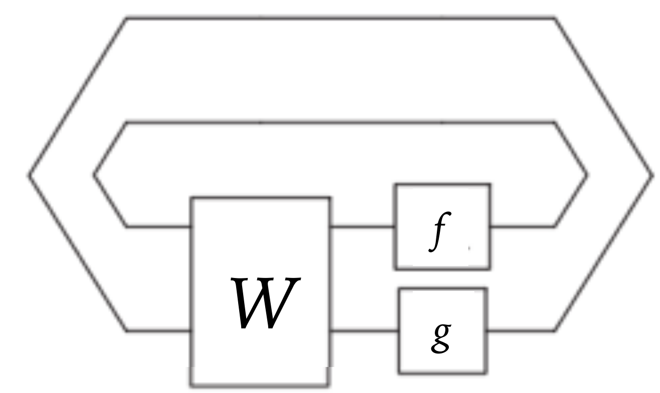
  Crucial difference: *linearity*.



- <u>Motivations to pose this question:</u>
  - Technical challenge; CTCs?
  - It is possible in the *classical* special case
  - Might help to characterize process matrices
  - Distinguish between violating and non-violating processes?
  - Challenge Mateus' challenge presented in his talk:
    Limits on the computational power

INTERMEZZO: THE CLASSICAL CASE

- Violations of causal inequalities is *not* a feature of quantum theory.
- With three parties or more: Classical violations possible.
- We know which processes are purifiable (can be made reversible)
- (Caution: Superluminal signaling without logical problems!)



- <u>Characterization:</u>
  $W: A \times B \rightarrow A \times B$ is a process iff
  $\forall\, f: A \rightarrow A,\ g: B \rightarrow B\ \exists!\ (x,y): (x,y) = W(f(x),g(y))$

  likewise for more parties.

Unique fixed point for every choice of *f*,*g*.

ÄB, S. Wolf, NJP 18, 1 (2016); ÄB, S. Wolf, NJP 18, 3 (2016)

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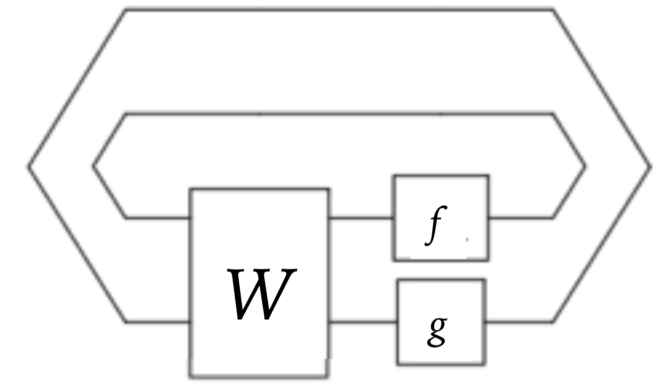


  likewise for more parties.

- <u>Interpretation:</u>
  Given the boundary conditions (W,f,g) states are *uniquely determined* *(fixed point)*.
  No grandfather antinomy (no overdetermination)
  No information antinomy (no underdetermination)

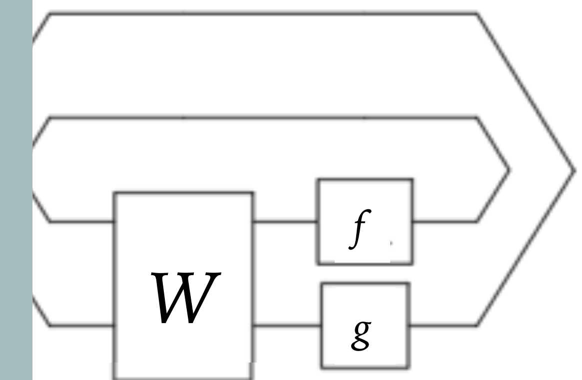
ÄB, S. Wolf, NJP 18, 1 (2016); ÄB, S. Wolf, NJP 18, 3 (2016)

INTERMEZZO: THE CLASSICAL CASE

- Characterizatio
  W: A × B → A
  ∀ f: A →A, g: B

  likewise for mo

- Interpretation:
  Given the boun … uniquely
  determined (fixed po
  No grandfather
  No information … n)

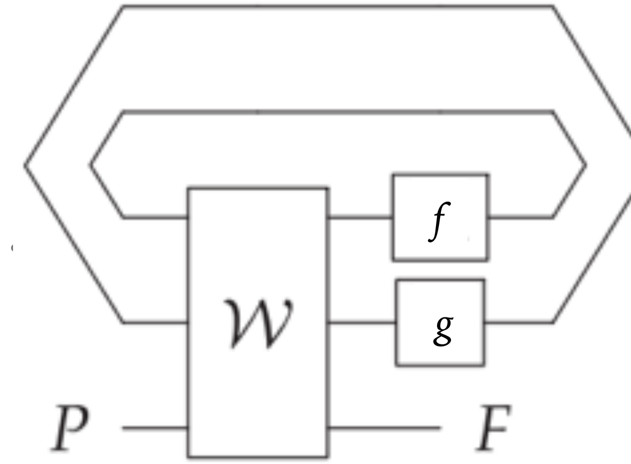


Three-party process (classical)

a,b,c	¬b∧c, ¬c∧a, ¬a∧b
0,0,0	0,0,0
0,0,1	1,0,0
0,1,0	0,0,1
0,1,1	0,0,1
1,0,0	0,1,0
1,0,1	1,0,0
1,1,0	0,1,0
1,1,1	0,0,0

ÄB, S. Wolf, NJP 18, 1 (2016); ÄB, S. Wolf, NJP 18, 3 (2016)

INTERMEZZO: THE CLASSICAL CASE



- <u>Characterization:</u>
  $W: A \times B \times C \rightarrow A \times B \times C$ is a process iff
  $\forall\, f: A \rightarrow A,\ g: B \rightarrow B,\ c\ \exists!\ x,y,z: (x,y,z) = W(f(x), g(y), c)$

  likewise for more parties.

- <u>Interpretation:</u>
  Given the boundary conditions (W,f,g,c) states are *uniquely determined* *(fixed point)*.
  No grandfather antinomy (no overdetermination)
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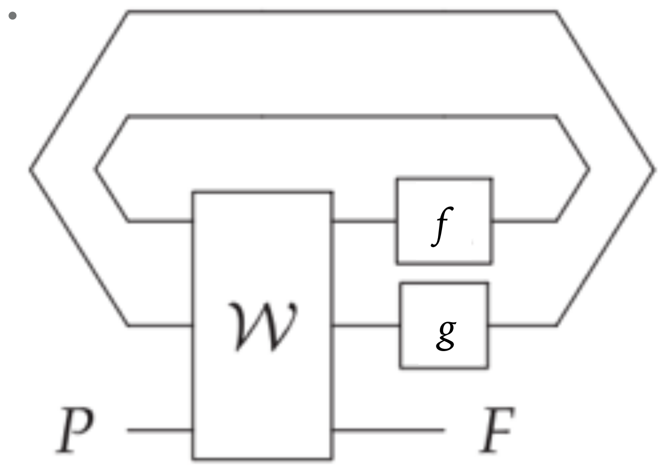
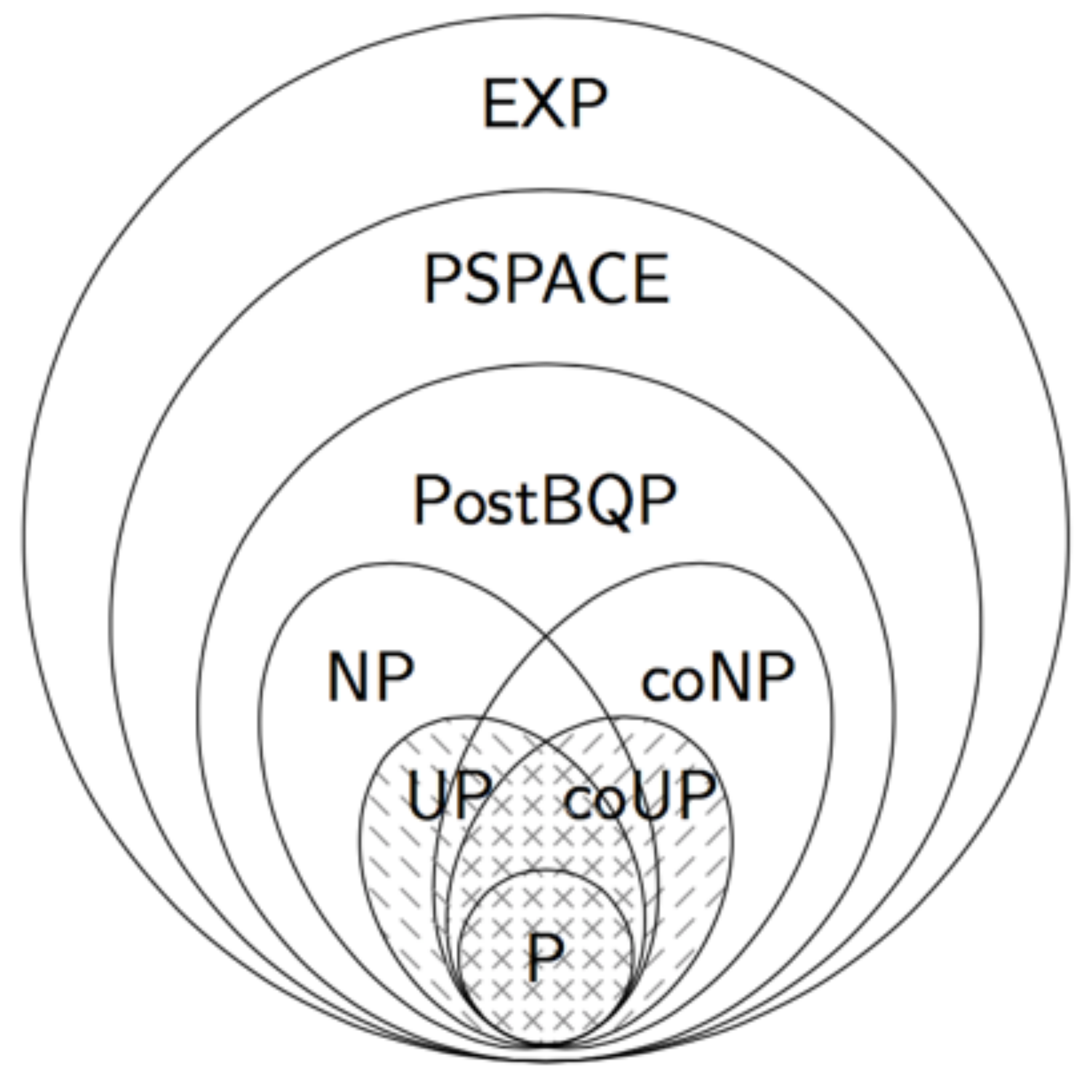
ÄB, S. Wolf, NJP 18, 1 (2016); ÄB, S. Wolf, NJP 18, 3 (2016)

INTERMEZZO²: THE CLASSICAL CASE AND COMPUTATION



- Helpful to upper bound the computational power of classical deterministic processes to UP ∩ coUP.

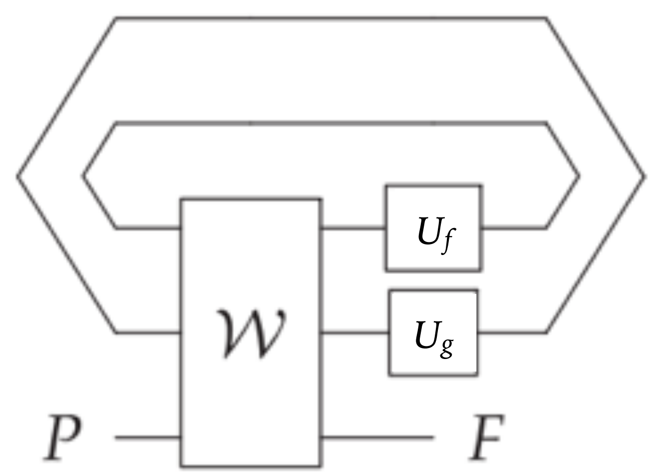
- Problems in UP ∩ coUP:
  - factoring
  - discrete log
  - parity games (quantum algorithm?)



ÄB, S. Wolf, PRSA 474, 2209 (2018)

BACK TO THE QUANTUM CASE

- Is there a *unique quantum* fixed point for every choice of U_f, U_g unitary and input (state at P)?

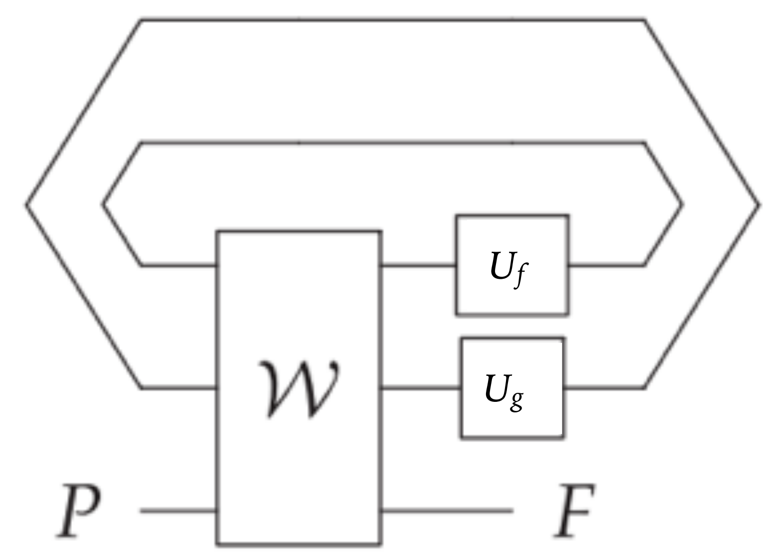


BACK TO THE QUANTUM CASE

- Observations:

  - Fixed points — if they exist — would be entangled with the input on *P*.
  (for different inputs there might be different fixed points)

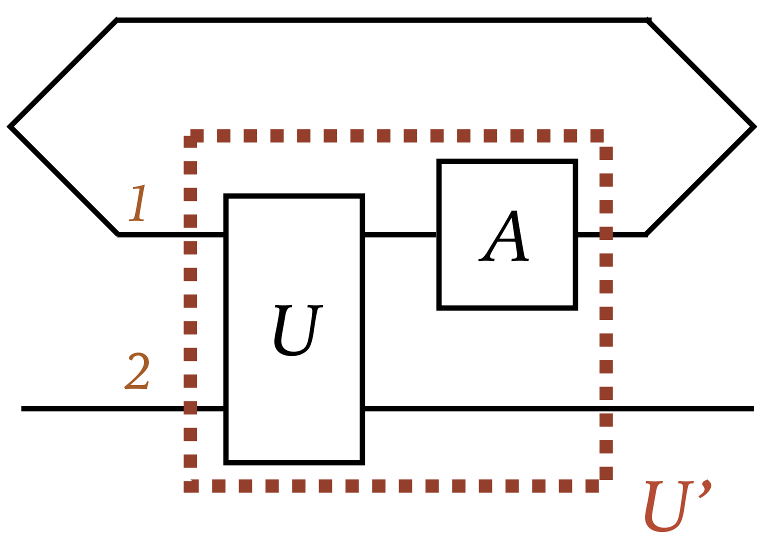
  - The process *W* might entangle the input on *P* with the rest!



QUANTUM „FIXED POINTS“

- Make use of superposition / entanglement

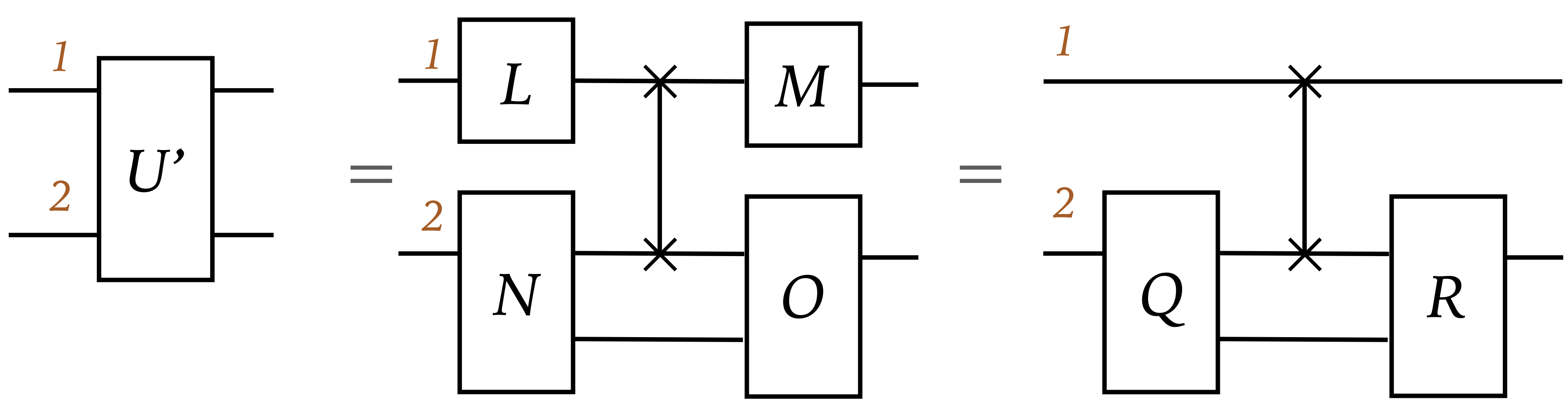
- <u>Single party:</u>



$= \mathrm{Tr}_1[(A \otimes I)U] = \mathrm{Tr}_1[U']$

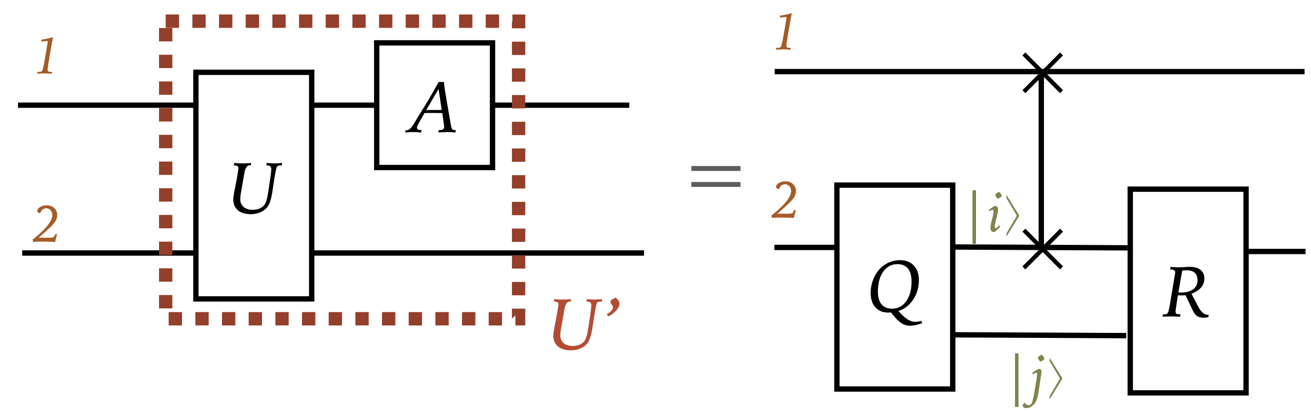
unitary

- We know the single-party characterization: States



QUANTUM „FIXED POINTS“

- What is the fixed point?



- <u>Ansatz:</u>
  There exists a basis $\{|b_{i,j}\rangle\}_{I\times J}$ such that
  $\forall\ i,j \in I\times J \quad \exists!\ x:\ U'|x\rangle|b_{i,j}\rangle = |x\rangle|b'_{i,j}\rangle$

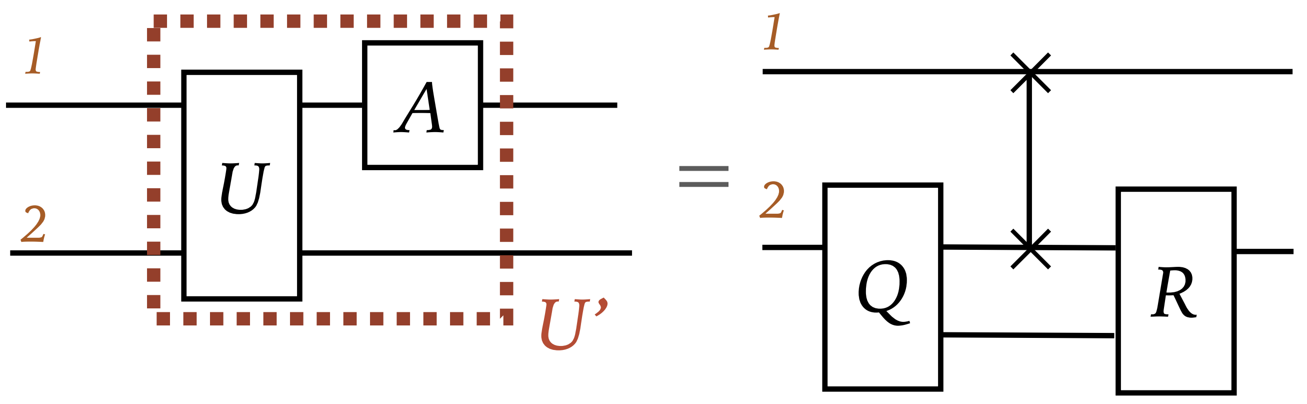
Computational basis (fixed)

- Easy to see: $|b_{i,j}\rangle = Q^{\dagger}|i,j\rangle$ and $x = i$.

- Description of the evolution $\mathrm{Tr}_1[U']$ for a basis $\{|b_{i,j}\rangle\}_{I\times J}$.

QUANTUM „FIXED POINTS" IN SUPERPOSITION

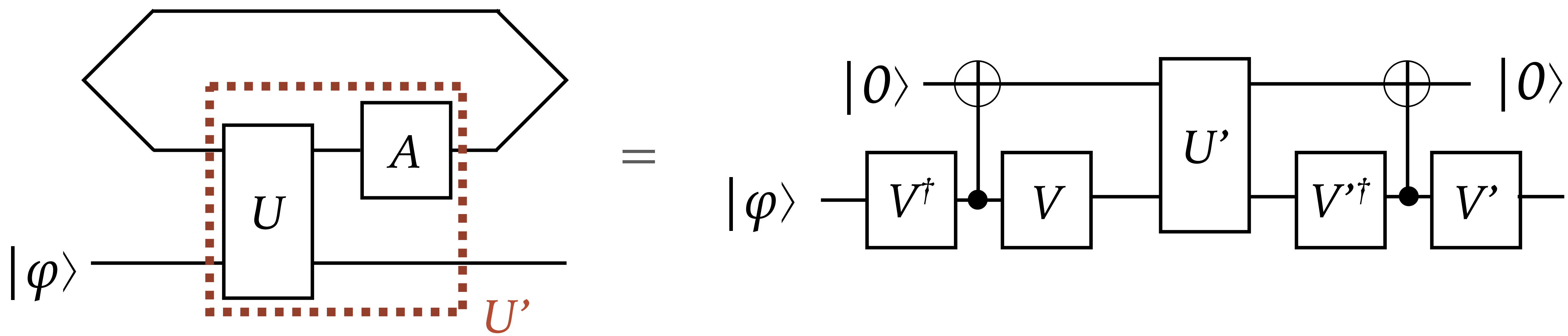
- Description of the evolution $\mathrm{Tr}_1[U']$ for a basis:
  $U'|i\rangle|b_{i,j}\rangle = |i\rangle|b'_{i,j}\rangle$



- For a general input $|\varphi\rangle$:
  1.) Express in $\{|b_{i,j}\rangle\}_{I\times J}$: $|\varphi\rangle = \sum_{i,j} \beta_{i,j}\, |b_{i,j}\rangle$
  2.) Entangle with respective fixed points: $\sum_{i,j} \beta_{i,j}\, |i\rangle|b_{i,j}\rangle$
  3.) Evolve through U': $\sum_{i,j} \beta_{i,j}\, |i\rangle|b'_{i,j}\rangle$
  4.) Disentangle from respective fixed points:
  $\sum_{i,j} \beta_{i,j}\, |b'_{i,j}\rangle = \mathrm{Tr}_1[U']|\varphi\rangle$

QUANTUM „FIXED POINTS" IN SUPERPOSITION

- Circuit picture



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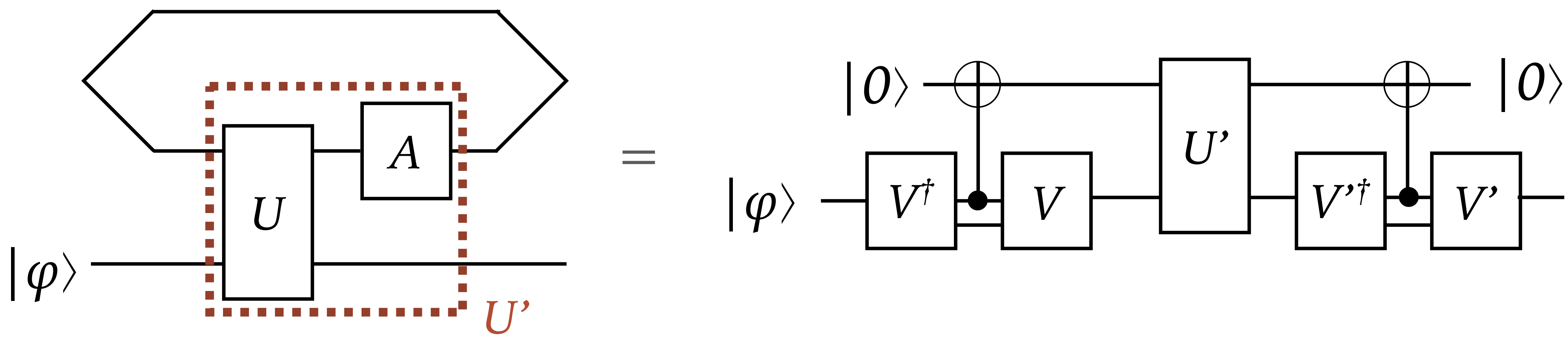
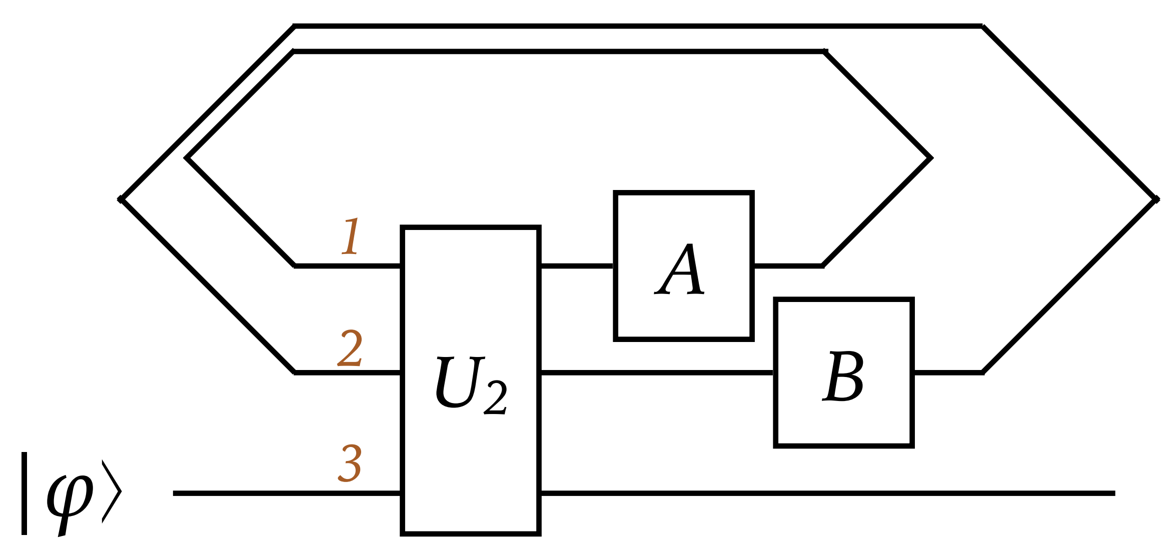
  $$\sum_{i,j} \beta_{i,j}\, |b'_{i,j}\rangle = \mathrm{Tr}_1[U']\,|\varphi\rangle$$

QUANTUM „FIXED POINTS" IN SUPERPOSITION

- Single party:

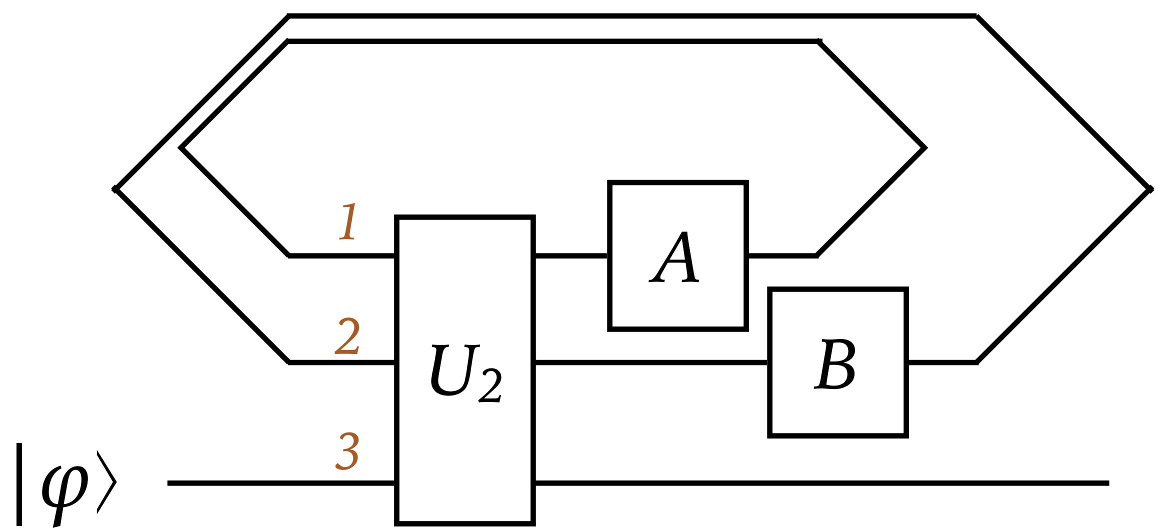


- Two parties: Apply recipe *recursively*.



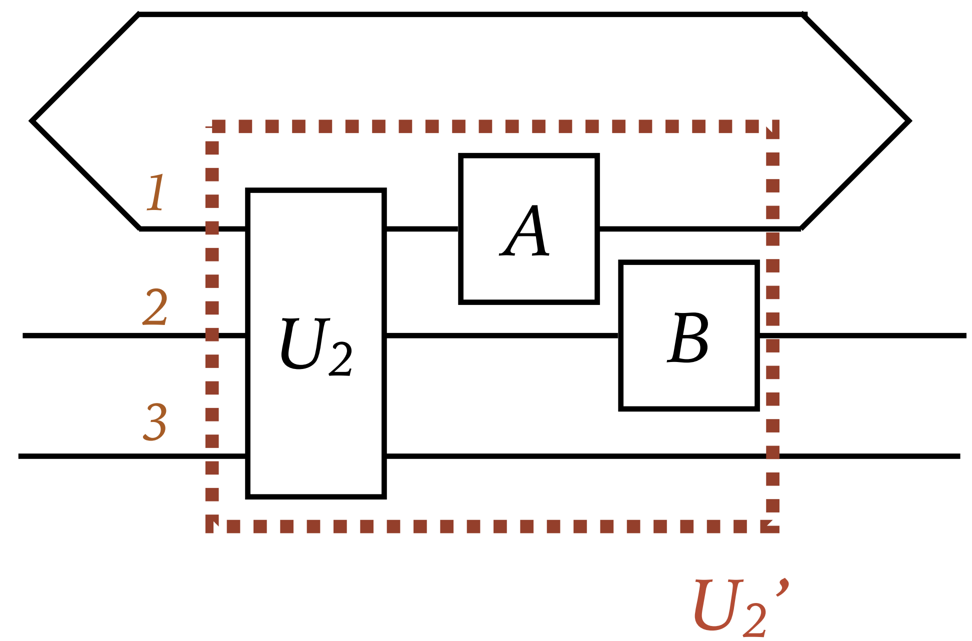
QUANTUM „FIXED POINTS" IN SUPERPOSITION

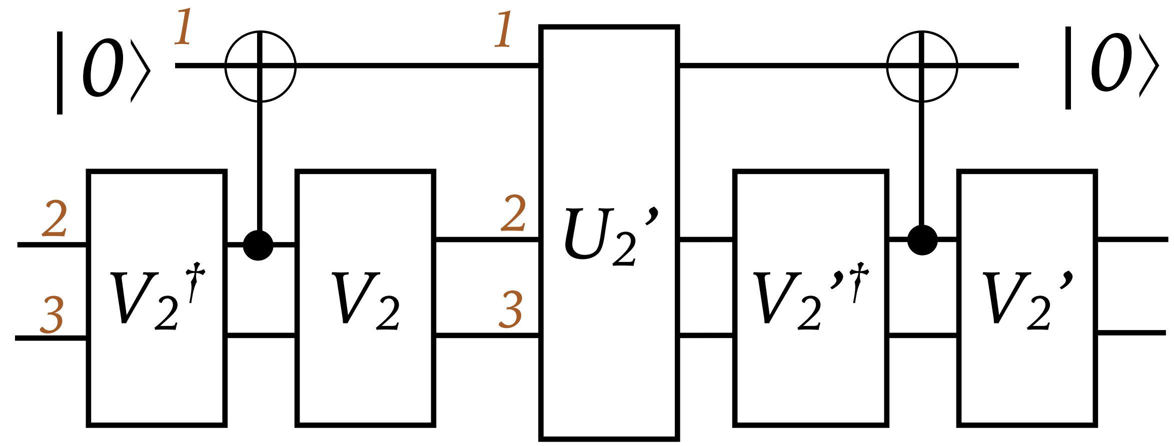
- Recursive application:



QUANTUM „FIXED POINTS" IN SUPERPOSITION

- <u>Recursive application:</u>
  1.) Contract Alice's CTC only
  (Single-party process)

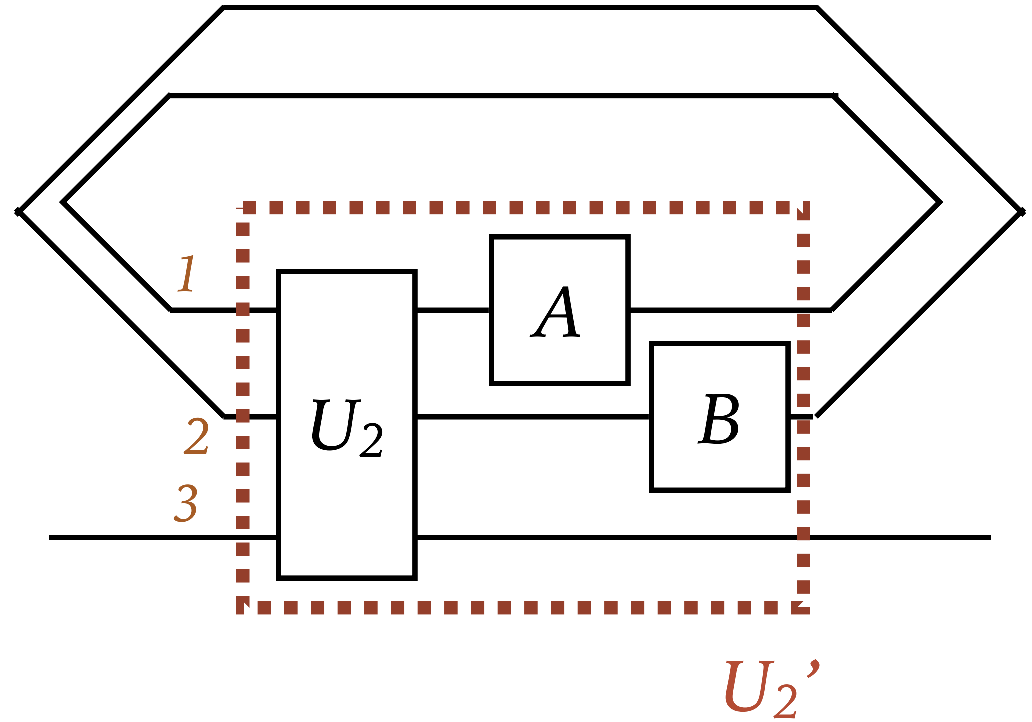


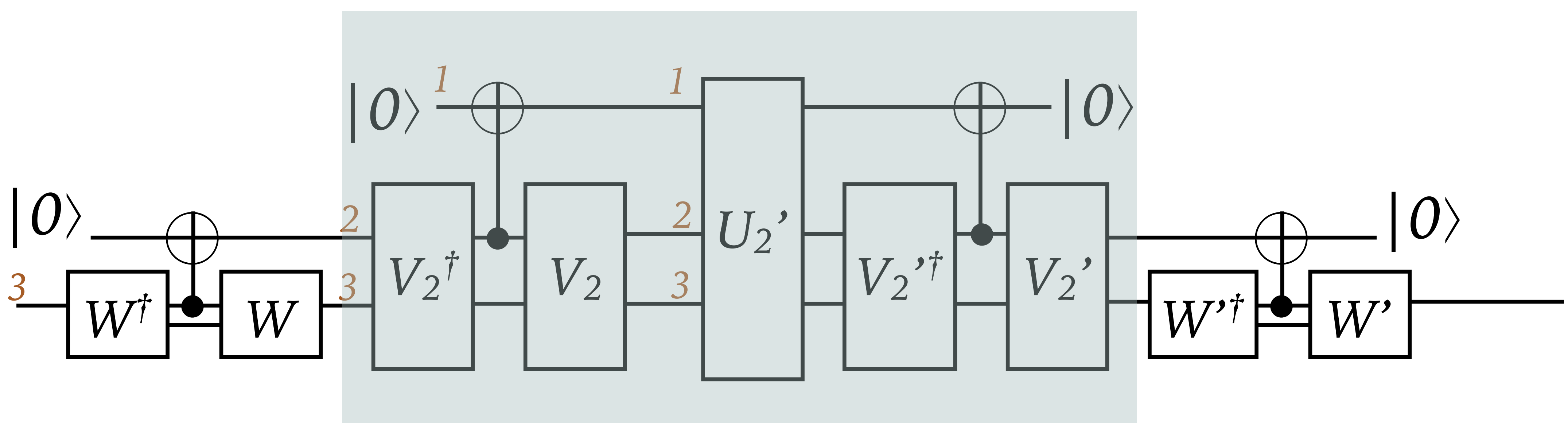


This implements the unitary $\mathrm{Tr}_1[U_2']$

QUANTUM „FIXED POINTS“ IN SUPERPOSITION

- Recursive application:
  1.) Contract Alice’s CTC only
  (Single-party process)
  2.) Contract Bob’s CTC
  (Single-party process)

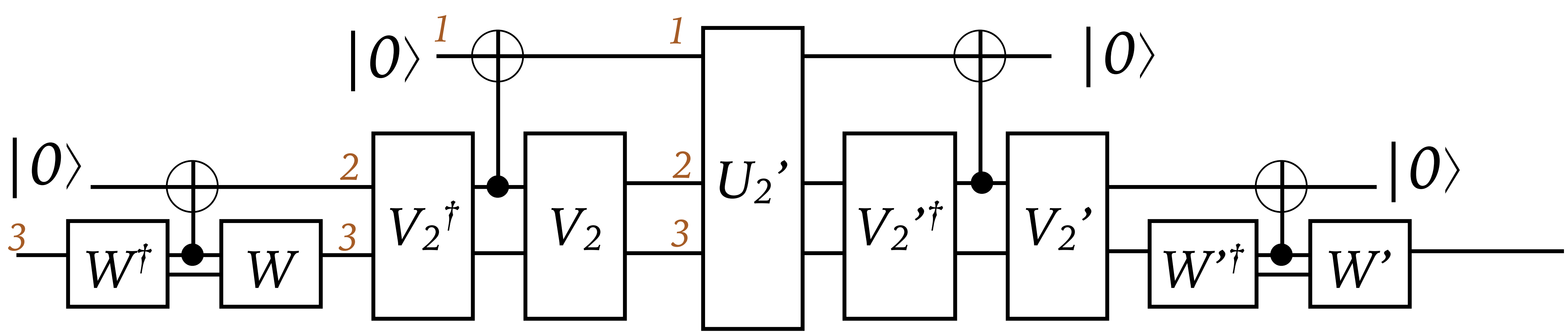




This implements the unitary $\mathrm{Tr}_{1,2}[U_2']$

QUANTUM „FIXED POINTS“ IN SUPERPOSITION

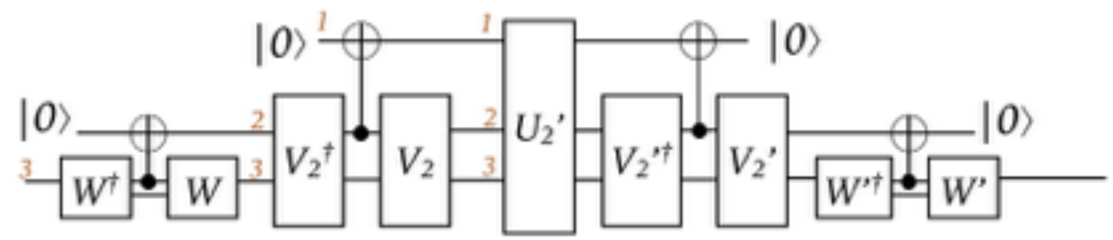
- For more parties:
  Continue recursively, contract one by one.



- What do we get?
  A state as input to $U_{2'}$ which describes all fixed points.

CHALLENGES

- Digest…



- Closed form instead of recursive application?
- What properties about the process can we read off the fixed points?
  Violations of causal inequalities?
- Simulations / Show computational limitations!
- Describe evolution in CTCs

GRAZIE MILLE